Corporate Finance, Module 3: Dividend growth model, practice problems

(The attached PDF file has better formatting.)

** Exercise 3.1: Dividend Growth Rate

A firm will have *earnings next year* of $7.50 per share. The firm *plows back* 80% of its earnings into projects with ROE's of 15%.

A. What is the dividend next year?
B. What is the *dividend growth rate*?
C. What is the book value per share next year?
D. What is the book value per share one year later?
E. What are the earnings one year later?
F. What is the dividend one year later?

Part A: The dividend next year is $7.50 × (1 – 80%) = $1.50.

Part B: The *dividend growth rate* is the *return on book equity* (ROE) times the *plow back* ratio:

$$80\% \times 15\% = 12.00\%$$

Note: Some final exam problems give the payout ratio. Know the relations:

- Plowback ratio = 1 – payout ratio.
- Dividend per share = earnings per share × payout ratio.
- Dividend growth rate = return on book equity × plow-back ratio.

Note: The rest of this exercise shows the logic for the dividend growth rate. The math is simple.

Part C: The book value per share is $7.50 / 15% = $50.

Part D: The new book value per share one year later is the book value next year plus the retained earnings:

$$\$50 + (\$7.50 – \$1.50) = \$56.$$

Part E: A year later, the earnings are 15% × $56 = $8.40.

Part F: The dividend one year later is $8.40 × (1 – 80%) = $1.680.

The dividend growth rate is $1.680 / $1.500 – 1 = 12.00%.

Question: Is the stock price more or less than $50 a share next year?

Answer: The stock price depends on the market capitalization rate for the stock. If this rate is more than 15%, the stock price is less than $50; if it is less than 15%, the stock price is more than $50.

** Exercise 3.2: Market Capitalization Rate

A stock trades at $74. The firm's return on book equity is 17%, its *payout ratio* is 45%, and its expected dividend next year is $4.20.

A. What is the firm's dividend yield as next year's dividend divided by the current stock price?
B. What is the firm's dividend growth rate?
C. What is the firm's market capitalization rate?

Part A: The dividend yield = $4.20 / $74 = 5.68%.

Part B: The dividend growth rate = the earnings growth rate = 17% × (1 – 45%) = 9.35%.

Part C: The dividend growth model says

$$P_0 = \frac{DIV_1}{(k-g)} \Rightarrow k = \frac{DIV_1}{P_0} + g$$

The market capitalization rate *k* is 5.68% + 9.35% = 15.03% ≈ 15%.

Brealey and Myers use *r*, not *k*, for the market capitalization rate; final exam problems use both symbols.

Know this relation: market capitalization rate = dividend yield + dividend growth rate.

Verify with the dividend growth model: $4.20 / (15% – 17% × 55%) = $74.34

** Exercise 3.3: Stock Price

A firm pays a dividend of $6 per share one year from now. If the firm's dividend growth rate is 4% per annum and its market capitalization rate is 12% per annum, what is the firm's expected stock price?

Solution 3.3: The dividend growth model says that $P_0 = DIV_1 / (k - g)$, where k is the market capitalization rate and g is the dividend growth rate.

- DIV_1 = $6.00
- k = 12.0%
- g = 4.0%

The expected stock price = P_0 = $6 / (12% – 4%) = $75.00

Note: Final exam problems may give the risk-free rate, the market risk premium, and the stock's beta, from which you derive the stock's market capitalization rate. They may give the earnings per share and either the payout ratio or the plow-back ratio, from which you derive the dividend next year. They may give the return on book equity (ROE) and the either the payout ratio or the plow-back ratio, from which you derive the dividend growth rate.

** Exercise 3.4: Stock Price

A firm will have *earnings next year* of $1.50 per share. Its *payout ratio* is 40%, its dividend growth rate is 9%, and its market capitalization rate is 11%.

A. What is the expected dividend next year?
B. What is the expected stock price now from the dividend growth model?

Part A: The expected dividend next year is 40% × $1.50 = $0.60.

Part B: The dividend growth model is $P_0 = DIV_1 / (k – g)$: $0.60 / (11% – 9%) = $30.00

** Exercise 3.5: Book Equity and Market Value

A firm has *book equity* of $60 per share, a *return on book equity* (*next year's* earnings divided by *current book equity*) of 14%, a payout ratio of 50%, and a market capitalization rate of 19%.

A. What are expected earnings next year?
B. What is the expected dividend next year?
C. What is the plow-back ratio?
D. What is the dividend growth rate?
E. What is the market value of the shares?

Part A: The expected earnings next year are $60 × 14% = $8.40.

Part B: The expected dividend is $8.40 × 50% = $4.20.

Part C: The plow-back ratio is the complement of the payout ratio: 1 – 50% = 50%.

Part D: The dividend growth rate is the return on book equity times the plow-back ratio: 14% × 50% = 7%.

Part E: The stock price is $4.20 / (19% – 7%) = $35.00.

Note: In the discussion of the dividend growth model, Brealey and Myers use ROE to mean the return on book equity, not the return on market value.

** Exercise 3.6: Stock Price

On January 1, 20X1, investors capitalize a firm with $100 million of cash, and the firm issues 1 million shares of stock.

The firm expects to earn 20% of book value for the next five years and to pay no dividends during this time. Its market capitalization rate for these five years is 20%.

After five years, the firm expects to earn 15% of book value and to pay half its earnings as dividends. Its market capitalization rate after five years will be 15%.

What is the market value of a share of this firm's stock on January 1, 20X1, from the dividend growth model?

Solution 3.6: For five years, the firm's expected earnings equal its market capitalization rate of 20%, and then earnings equal the new market capitalization rate of 15%. The present value of growth opportunities is zero, so the market value of the stock equals its book value of $100.

Know the relations: If the firm's return on book equity is more than its market capitalization rate, its present value of growth opportunities is positive. If its return on book equity is less than its market capitalization rate, its present value of growth opportunities is negative. It should return all earnings to shareholders, not plow back any money into the firm.

** Exercise 3.7: Stock Price

A firm with operations in South-East Asia has a book equity per share of $100. It earns a 15% return on book equity and it pays out 40% of its earnings in dividends next year. The risk-free rate is 8% per annum, the market risk premium is 10%, and the stock's CAPM beta is 1.200.

Because of rising hostilities in South-East Asia, the market risk premium for firms with operations there rises to 15%. All other figures remain the same.

A. What is the dividend next year?
B. What is the dividend growth rate?
C. What is the firm's stock price before the change in the market risk premium?
D. What is the firm's stock price after the change in the market risk premium?
E. What is the percentage change in the firm's stock price?

Part A: The dividend is $100 × 15% × 40% = $6.

Part B: The dividend growth rate is 15% × 60% = 9%.

Part C: The old stock price is $6 / (20% – 9%) = $54.55

Part D: The new stock price is $6 / (26% – 9%) = $35.29.

Part E: The dollar change in the stock price is $35.29 – $54.55 = $-19.26.

The percentages change is –$19.26 / $54.55 = -35.31%

Know the effect of each variable on the firm's stock price. This exercise shows that an increase in the market risk premium (with no other changes) reduces the firm's stock price.

** Exercise 3.8: Stock Price

Investors capitalize a firm with $80 million of cash, and the firm issues 2 million shares of stock. The firm expects to earn 25% of book value for the next five years. To conserve its cash, the firm expects to pay a 25% stock dividend but no cash dividend. Its market capitalization rate for these five years is 25%.

For the next five years, the firm expects to earn 12% of book value and to pay all of its earnings as dividends. Its market capitalization rate after five years will be 12%.

A. What is the firm's expected market value of a share of stock at inception?
B. What is the firm's expected market value of a share of stock at the end of the *first* five years?
C. What is the firm's expected market value of a share of stock at the end of the *second* five years?

Part A: The firm's expected earnings each year equal its capitalization rate, so its stock price at inception equals the initial book value of share, or $80 million / 2 million = $40.

Part B: During the first five years, the stock's value increases 25% a year, which is just offset by the stock dividend. In the first year, each stock increases from $40 to $50. The stock is then split into one and a quarter shares, worth $40 + $10. At the of five years, more shares are outstanding, but each share is worth $40.

Part C: During the second five years, the firm earns just enough to satisfy investors. All earnings are paid as dividends, so the stock price remains $40.

** Exercise 3.9: Payout Ratios

A firm's projects earn 20% of book equity, which is $100 per share. The firm's market capitalization rate is 20%.

A. What is the market value of the firm's stock at a 40% payout ratio?
B. What is the market value of the firm's stock at a 60% payout ratio?
C. Explain the relation of these two values.

Part A: The dividend growth rate is the return on book value times the plow-back ratio, which is one minus the payout ratio. The annual earnings are the *return on book value* times *book value* per share, which is $100. The dividend is the annual earnings times the payout ratio.

- The dividend growth rate is 60% × 20% = 12%.
- The stock price is (20% × $100 × 40%) / (20% – 12%) = $100.

Question: The exercise says the stock is priced at $100 a share. What did we solve?

Answer: The exercise gives the book value of shares. We solved for the market value of shares.

Part B: The dividend growth rate = 40% × 20% = 8%. The stock price = (20% × $100 × 60%) / (20% – 8%) = $100.

Part C: The return on book equity equals the market capitalization rate, so the dividend yield has no effect on the firm's stock price.

Question: Isn't it always true that the dividend yield does not affect the stock price? Isn't this one of the Miller and Modigliani propositions?

Answer: This should be true in *perfect capital markets*, where the tax rate is zero, the firm invests in zero NPV projects, and the dividend yield does not convey information to investors.

Question: What happens if the firm invests in positive NPV projects?

Answer: If the firm invests in positive NPV projects, the return on book equity is higher than the capitalization rate, and the firm is profitable. If markets are competitive, other firms enter the market and compete away the excess profits. In the long-run, each firm's profits exactly match its risk.

If the return on book equity does not equal the market capitalization rate, dividend policy matters. Consider two firms with a book value per share of $100 and a market capitalization rate of 41%.

- Firm Y pays 20% of earnings as dividends.
- Firm Z pays 80% of earnings as dividends.

Each firm earns a 50% return on book equity. The dividend growth rates are

- Firm Y: 80% plow-back ratio × 50% ROE = 40%
- Firm Z: 20% plow-back ratio × 50% ROE = 10%

Next year's dividend per share is

- Firm Y: 20% payout ratio × $50 = $10
- Firm Z: 80% payout ratio × $50 = $40

The expected stock prices are

- Firm Y: $10 / (41% – 40%) = $1,000
- Firm Z: $40 / (41% – 10%) = $129

In competitive markets, the return on book equity reflects the systematic risk of the firm. Other firms enter the market and drive down the return on book equity. This scenario assumes capital markets are not perfect.

The expected stock price is given by the dividend growth model only if the firm operates optimally. Suppose a firm has a monopoly with a high demand for its products.

If the firm earns a return on book equity greater than its market capitalization rate, it should pay low dividends (or no dividends). If the firm pays high dividends and does not allow its owners to contribute more capital, it is not giving its owners the profits they could get from growth of the firm. In a perfect capital market, the shareholders would replace the present management with other managers. If they can not do this, the stock price declines.

Know how the payout ratio and the difference between the return on book equity and the market capitalization rate affect the stock price.