Corporate Finance, Module 4: "*Net Present Value vs Other Valuation Models*"

(Brealey and Myers, Chapter 5)

Practice Problems

(The attached PDF file has better formatting.)

Question 4.1: Accounting Returns

{*Note:* The final exam does not cover the payback method, discounted payback method, or the return on book equity. Statement B is relevant to the final exam for this course.}

Which of the following is true?

A. Accounting methods affect the average return on book value.
B. A borrower should accept a proposal to borrow money if the internal rate of return is greater than the opportunity cost of capital.
C. The average return on book value method recognizes that immediate receipts are more valuable than distant receipts.
D. The discounted payback method is equivalent to the NPV method.
E. None of A, B, C, or D is true.

Answer 4.1: A

Statement A: The depreciation schedule and inventory method (LIFO vs FIFO) affect the book value in each year, the average book value, the accounting income in each year, and the return on average book value.

Question: Insurance doesn't have much depreciation or inventory. What accounting methods affect return on book equity for insurance?

Answer: The major items are amortization of deferred policy acquisition costs, loss reserves and policy reserves, treatment of deferred tax assets and liabilities, and capitalization of post-retirement pension and medical costs. When we deal with insurance cash flows, we must consider the statutory accounting rules, since distributable income depends on statutory accounting income, not the insurer's cash flows with policyholders and claimants. These issues are discussed on SOA Course 5 and CAS Exam 9, not here.

Statement B: Borrowing differs from investing:

- A proposal to *invest* money should be accepted if the IRR is greater than the opportunity cost of capital.
- A proposal to *borrow* money should be accepted if the IRR is *less* than the opportunity cost of capital.

Question: If a loan is a positive net present value project for the bank, it must be a negative NPV project for the borrower, and vice versa. Can a loan ever be a positive NPV project for both parties?

Answer: Although the loan has the same IRR for the two parties, the two parties have different opportunity costs of capital. Suppose a person wants a car loan, and the bank offers a 12% rate. With the probability of default, the expected return for the bank is 9%.

- If the bank's next best use of the funds is an 8% loan to a firm, the car loan is a positive NPV project.
- If the borrower's next best source of funds is a 13% loan from a credit union, the bank loan is a positive NPV project.

Statement C: The average return on book value uses nominal dollar accounting entries, with no adjustment for the time value of money.

Statement D: The discounted payback method considers when the present value of the cash inflows first exceeds the present value of the cash outflows; it does not consider the total value of all cash inflows, including subsequent cash flows. Consider 2 projects:

	Time 0	*Time 1*	*Time 2*
Project 1	-100	+120	+60
Project 2	-100	+90	+120

At a discount rate of 10%, the discounted paybacks are 1 year for Project 1 and 2 years for Project 2. If we interpolate between years, the payback periods are a bit less than 1 year for Project 1 and a bit more than 1 year for Project 2.

The net present values are

- Project 1: $-100 + 120 / 1.1 + 60 / 1.1^2 = \$58.68$
- Project 2: $-100 + 90 / 1.1 + 120 / 1.1^2 = \$80.99$

Exercise 4.2: Depreciation Methods

We have a choice between straight line depreciation and accelerated depreciation. Which method gives a higher average return on book value?

{*Note:* This practice problem is designed to show the effects of depreciation. The financial accounting course covers depreciation schedules; this problem shows how to deal with depreciation in determining net present value. As you read this practice problem, focus on how depreciation is included in the cash flows.}

Solution 4.2: Book income is the numerator of the return on book value; book value is the denominator.

- The *average* book income is the same under either method of depreciation, though the timing of income recognition differs.
- The average book value is higher if the depreciation is delayed longer.

The average return on book value is lower if depreciation is delayed longer.

Straight line depreciation is a slower process than accelerated depreciation, so straight line depreciation gives a *higher* average *book value* and a *lower* average *return on book value*.

Illustration: Suppose a project requires $5,000 of working capital, which is not depreciated, and a $1,000 machine. The project lasts two years, and the machine is depreciated over 1 year (accelerated depreciation) or two year (straight line depreciation). The machine has no salvage value. Revenue before depreciation is $1,200 each year.

Option A: If the machine is depreciated over 1 year (accelerated), we have

Year	*Equity (End of Year)*	*Average Equity*	*Depreciation*	*Revenue (Pre-depreciation)*	*Earnings*	*Return on Book Value*
0	6,000					
1	5,000	5,500	1,000	1,200	200	3.64%
2	5,000	5,000	0	1,200	1,200	24.00%
Average						13.82%

Option B: If the machine is depreciated over 2 years (straight line), we have

Year	*Equity (End of Year)*	*Average Equity*	*Depreciation*	*Revenue (Pre-depreciation)*	*Earnings*	*Return on Book Value*
0	6,000					
1	5,000	5,750	500	1,200	700	12.17%
2	5,000	5,250	500	1,200	700	13.33%
Average						12.75%

Quicker depreciation leads to lower book value and a higher return on book value. But the depreciation schedule is an accounting item that should not affect the profitability measure.

We used a straight average for the average return on book value. Had we used a weighted average, where the weights are the average equity in each year, the average return on book value would be

- Option 1: ($200 + $1,200) / ($5,500 + $5,000) = 13.33%

- Option 2: ($700 + $1,700) / ($5,750 + $5,250) = 12.73%

We used average equity during the year for this problem. Some problems use beginning of the year equity for the denominator.

{*Note:* For the final exam, we focus on the NPV and IRR measures of profitability. Tax depreciation schedules are relevant, since they affect the cash flows; book depreciation schedules, such as GAAP depreciation schedules, are not relevant.}

Exercise 4.3: Capitalization Rates

We have a choice between a 10% and a 12% capitalization rate for a three year project, with a single investment at time zero and positive cash flows in each subsequent year. All cash flows occur at the end of the year.

A. Which capitalization rate gives a higher net present value for the project?
B. Which capitalization rate gives higher economic income in the third year?

Solution 4.3:

Part A: Since the cash inflows occur after the cash outflow (the investment), the lower capitalization rate of 10% gives the higher net present value.

Let the cash flows be CF_0, CF_1, CF_2, and CF_3, where $CF_j > 0$ for j = 0, 1, 2, 3.

- NPV at 10% = $-CF_0 + CF_1 / 1.10 + CF_2 / 1.10^2 + CF_3 / 1.10^3$.
- NPV at 12% = $-CF_0 + CF_1 / 1.12 + CF_2 / 1.12^2 + CF_3 / 1.12^3$.

The NPV at 10% minus the NPV at 12% =

$$CF_1 \times (1/1.10 - 1/1.12) + CF_2 \times (1/1.10^2 - 1/1.12^2) + CF_3 \times (1/1.10^3 - 1/1.12^3) > 0,$$

since the cash flows in years 1, 2, and 3 are positive.

Question: Could you summarize this principle?

Answer: We divide projects into investments vs loans. A firm investing has cash outflows before cash inflows; a firm borrowing has cash inflows before cash outflows.

For an investment, the longer we must wait for the cash inflows, the lower the net present value of the project. The higher the capitalization rate, the lower the net present value. If interest rates rise and the projected cash flows do not change, the net present value of the project declines.

For a loan, if interest rates rise after the loan is taken, the net present value of the loan increases. Most problems on the final exam discuss the present value of the tax shields from the debt payments or the present value of subsidized financing. We cover these topics in later modules.

Part B: If the cash flow at the end of the third year is Z and the capitalization rate is k, the worth of the project at the beginning of the third year is $Z / (1 + k)$. The economic income during the third year is the capitalization rate times the worth of the project at the beginning of the year, or $k \times Z / (1 + k)$.

This expression is strictly increasing in k. The Z is a constant, so we ignore it; we show that $k / (1 + k)$ is strictly increasing in k. We do this two ways:

- The partial derivative of this expression with respect to k is $1 / (1 + k) - k / (1 + k)^2$. Since $1 / (1 + k)$ is positive, the sign of this derivative is the same as $1 - k / (1 + k) > 0$.
- The reciprocal of this expression is $1 + 1/k$, which is strictly decreasing in k, so the first expression is strictly increasing in k.

A higher capitalization rate causes higher economic income during the last year. That is, a higher capitalization rate causes a lower net present value at the beginning of the year and higher economic income during the year.

Question: This seems contradictory. A higher capitalization rate causes higher economic income but a lower net present value; how can this be?

Answer: For GAAP statements, income occurs in periods. If a five year project is worth $50,000, GAAP may show income of $10,000 each year. In contrast, the economic worth of a project is recorded at inception. If a five year project has a net present value of $50,000, the entire $50,000 is shown at inception. The subsequent economic income is not the worth of the project; it is the return using an economic accounting framework.

Exercise 4.4: Equipment Cost

Two firms have five year identical projects and depreciation schedules. Firm A buys the equipment for $X million and Firm B buys the equipment for $Y million, with X > Y.

A. Which firm has the higher average return on book value?
B. Which firm has the higher net present value?
C. Which firm has the higher economic income in year 3, if we do not consider tax depreciation?
D. Which firm has the higher economic income in year 3, if we consider tax depreciation?

{*Note:* The firm which spends less to buy the equipment is more profitable. The practice problem shows that average return on book value is not a reasonable profitability measure, since it may give the opposite result. Similarly, economic income is not a measure of profitability; we use net present value, not economic income.}

Solution 4.4:

Part A: Firm B, with the lower costs for the equipment, has lower depreciation charges and higher net income. This is somewhat offset by the higher equity for the project in Firm B, but the average return on book value remains higher in Firm B.

Illustration: Suppose X is $2 million, Y is $1 million, both firms use $3 million of working capital, and revenue before depreciation is $1 million a year.

- *Firm A:* Book value declines from $5 million to $3 million, for an average of $4 million.
- *Firm B:* Book value declines from $4 million to $3 million, for an average of $3.5 million.

Depreciation is $400,000 a year for Firm A and $200,000 a year for Firm B, so revenue after depreciation is $600,000 a year for Firm A and $800,000 a year for Firm B.

Average return on book value is about $600,000 / $3 million = 20% for Firm A and about $800,000 / $3.5 million = 22.86% for Firm B (exact amount depends on type of average).

Part B: Firm A has the greater cash *outflow* to buy the equipment, so it has the lower net present value. In the illustration, both firms have the same cash *inflows* for five years. Firm A has $1 million higher outflow at inception, so its net present value is $1 million lower.

Part C: The economic income depends on cash flows in year 3 and subsequent; it does not depend on the cost of the equipment before time 3. Both firms have the same economic income in year 3 if they have the same tax write-off each year.

Part D: The tax write-off is 35% of the depreciation charge, which is higher for Firm A. Since Firm A has greater tax write-offs each year, it has the higher economic income in year 3.

Question: The textbook does not discuss tax depreciation; must we know this subject?

Answer: This chapter (this module) does not discuss tax depreciation. Later modules discuss this subject in detail.

Question: How would we summarize the principle shown here?

Answer: The GAAP accounting principle is that we defer recognition of income over the term of the project. Economic income is the opposite. We recognize the net present value of the project at inception. The economic income in subsequent years is just the cost of holding capital; it is unrelated to the value of the project.

Question: How does risk affect the net present value and the economic income?

Answer: Greater systematic risk increases the capitalization rate, decreases the net present value of the project at inception, and increases the economic income in subsequent years.

Exercise 4.5: NPV and Economic Income

A firm has a five year project. Joseph, the firm's president, asks his two financial analysts, Ephraim and Menasheh, to evaluate the project using an NPV analysis. Ephraim and Menasheh agree on the expected cash flows, but they disagree on the capitalization rate. One of them recommends that the firm accept the project; the other recommends that the firm reject the project.

Joseph creates the following table:

	Ephraim	*Menasheh*
NPV		
Yr 1 Ec Inc		
Yr 2 Ec Inc		
Yr 3 Ec Inc		
Yr 4 Ec Inc		
Yr 5 Ec Inc	$10,000	$12,000

A. Which analyst has the higher Year 4 economic income?
B. Which analyst has the higher total economic income for the five years (not including the net present value)?
C. Which analyst recommends that the firm accept the project?

Solution 4.5:

Part A: Mehasheh has the higher economic income in year 5, so he is using the higher capitalization rate and he has the higher economic income in year 4 as well.

Question: Can you show the mathematics of this? Suppose Ephraim uses a 10% capitalization rate and all cash flows occur at year end.

Answer: A 10% capitalization rate and $10,000 of economic income means that the present value of future cash flows at the beginning of year 5 is $10,000 / 10% = $100,000, so the cash flow at the end of year 5 is $110,000. Menasheh shows $12,000 of economic income. If his capitalization rate is R, the present value of future cash flows at the beginning of year 5 is $110,000 / (1+R), and the economic income in year 5 is $110,000R/ (1+R) = $12,000. We solve for R:

$$\$110{,}000R/\,(1+R) = \$12{,}000 \Rightarrow$$
$$\$110{,}000R = \$12{,}000 \times (1 + R)$$
$$\$110{,}000R - \$12{,}000R = \$12{,}000$$
$$R = \$12{,}000 / \$98{,}000 = 12.24\%$$

Part B: By the same reasoning, Menasheh has higher economic income every year and a higher total value of the economic income over the five years.

Question: Is this just a higher gross economic income or also a higher net present value of the economic income?

Answer: In general, it is also a higher net present value of the economic income, but this depends on the cash flows. For the exercise, we consider just the gross economic income.

Part C: Menasheh has the higher capitalization rate and the lower net present value. He rejects the project,

whereas Ephraim accepts the project.

Exercise 4.6: Size of Project

{*Note:* The final exam will test this type of problem in a multiple choice question format.}

Suppose the opportunity cost of capital is 10% per annum. Use both net present value and internal rate of return to solve the problem below.

- Projects A and B have one year durations and the same initial investment of $10,000.
- Project A returns $11,100 and Project B returns $11,200 after one year.
- Project C has an initial investment of $20,000 and returns $22,300 after one year.

The cash flows of Project C are the sum of the cash flows of Projects A and B. The projects are mutually exclusive.

Part A: If a firm has a choice of Projects A, B, or C, which should it choose? Show the answer according to the net present value criterion and the internal rate of return criterion.
Part B: Why do the net present value and internal rate of return criteria give different answers?

{This exercise is adapted from a past Course 2 exam problem. Brealey and Myers say that the NPV recommendation is correct and the IRR recommendation is not correct.}

Solution 4.6: *Part A:* The net present values of the projects are

- Project A: –$10,000 + $11,100 / 1.10 = $90.91.
- Project B: –$10,000 + $11,200 / 1.10 = $181.82.
- Project C: –$20,000 + $22,300 / 1.10 = $272.73.

The firm should choose Project C, which has the highest net present value.

The internal rates of return of the projects are

- Project A: –$10,000 + $11,100 / (1 + irr) = 0 ⇛ irr = $11,100 / $10,000 – 1 = 11.00%.
- Project B: –$10,000 + $11,200 / (1 + irr) = 0 ⇛ irr = $11,200 / $10,000 – 1 = 12.00%.
- Project C: –$20,000 + $22,300 / (1 + irr) = 0 ⇛ irr = $22,300 / $20,000 – 1 = 11.50%.

The firm should choose Project B, which has the highest net present value.

Part B: The NPV criterion assumes that other projects available to the firm give a return of 10%, the opportunity cost of capital. All other projects have zero NPV.

- If the firm has $20,000 to invest, and invests in Project A plus another project returning the opportunity cost of capital, the total net present value is $90.91.
- Similarly, if the firm invests in Project B plus another project returning the opportunity cost of capital, the total net present value is $181.82.

The IRR criterion implicitly assumes that other projects of the firm have the same opportunity cost of capital as the project being considered. This implies that the firm can invest in as many projects like Projects A, B, or C as it desires.

- If the firm has $20,000 to invest and choose two projects like Project A, the total net present value is 2 × $90.91 = $181.82.
- If the firm chooses two projects like Project B, the total net present value is 2 × $181.82 = $363.64.

Brealey and Myers assume that the net present value assumption is correct. When pricing insurance products, actuaries often assume that the internal rate of return assumption is correct. Both the SOA syllabus

(Atkinson and Dallas textbook on Course 5) and the CAS syllabus (Feldblum study note on Exam 9) assume the IRR assumption is correct.

The final exam will not ask why NPV is better than IRR (or vice versa). It may ask about the assumption in each criterion about the expected return of other projects.

Question: Why do actuaries use IRR when Brealey and Myers recommend NPV?

Answer: The actuary may compare the profitability of a workers' compensation policy with that of an auto insurance policy. The company may have a choice of writing only one of the two lines of business, if it does not have the facilities to write both. But it evaluates the profitability of a policy, not of its block of business. If auto insurance is profitable, it may write 1,000 policies, and it expects the same return on each policy. This is the IRR assumption, not the NPV assumption.

Question: Do you mean that IRR is correct for actuaries, and NPV is not?

Answer: We must examine each scenario to see whether the NPV assumption or the IRR assumption is more reasonable.

Question: How would Brealey and Myers respond to the viewpoint of these actuaries?

Answer: Brealey and Myers say that the capitalization rate is the opportunity cost of capital, which is the rate earned on the next best project. If the firm can earn 11% or 12% writing personal auto or workers' compensation policies, then its opportunity cost of capital is 11% or 12%, not 10%. Since we say the capitalization rate is 10%, we mean that the firm can not earn 11% or 12% writing other policies.

Question: How would the actuaries respond to Brealey and Myers?

Answer: We don't treat each policy as a separate project. We price a line of business, not an individual policy. The opportunity cost of capital is the return on the next best type of business, such as running a bank instead of an insurance company.

Question: How would Brealey and Myers respond to this argument?

Answer: If this is the scenario, the exercise should be worded differently. Instead of saying the firm can invest $10,000 or $20,000, the exercise should say that the firm can invest an unlimited amount K of capital. The argument not whether NPV or IRR is better; we all agree that NPV is better. The argument is about the return on unused capital.

Question: Suppose a firm has an unlimited number of projects, all earning the same return, which is more than shareholders could receive from other businesses. If the opportunity cost of capital is the return on the next best use of capital, the NPV is zero, since the firm has an unlimited number of projects with this return.

Answer: In practice, we define the opportunity cost of capital as the return that suppliers of capital would receive in the next best use of their funds. If the insurer earns 12% on the capital, and its shareholders would earn only 10% in other businesses, the capitalization rate is 10%. Similarly, if the firm's managers are exceedingly gifted and can earn 12% on any business project, but the firm's shareholders can get only 10% in other businesses, the opportunity cost of capital is 10%.

Exercise 4.7: Length of Project

Suppose the opportunity cost of capital is 10% per annum. Use both net present value and internal rate of return to solve the problem below.

- Projects A and B have one year durations and the same initial investment of $10,000.
- Project A returns $11,100 and Project B returns $11,200 after one year.
- Project C has an initial investment of $10,000 and returns $11,100 after one year (like Project A), but then re-invests this $11,100 and returns 1.11 × $11,200 = $12,432.00 after two years (like Project B). In sum, Project C has an initial investment of $10,000 and returns $12,432 after two years.

The projects are mutually exclusive.

Part A: If a firm has a choice of Projects A, B, or C, which should it choose? Show the answer according to the net present value criterion and the internal rate of return criterion.
Part B: Why do the net present value and internal rate of return criteria give different answers?

{*Note:* The final exam will test this type of problem in a multiple choice question format.}

Solution 4.7: *Part A:* The net present values of the projects are

- Project A: –$10,000 + $11,100 / 1.10 = $90.91.
- Project B: –$10,000 + $11,200 / 1.10 = $181.82.
- Project C: –$10,000 + $12,432 / 1.10^2 = $274.38.

The firm should choose Project C, which has the highest net present value.

The internal rates of return of the projects are

- Project A: –$10,000 + $11,100 / (1 + irr) = 0 ⇛ irr = $11,100 / $10,000 – 1 = 11.00%.
- Project B: –$10,000 + $11,200 / (1 + irr) = 0 ⇛ irr = $11,200 / $10,000 – 1 = 12.00%.
- Project C: –$10,000 + $12,432 / $(1 + irr)^2$ = 0 ⇛ irr = ($12,432/120,000)^{½} – 1 = 11.50%.

The firm should choose Project B, which has the highest net present value.

Part B: The NPV criterion assumes that other projects available to the firm give a return of 10%, the opportunity cost of capital. All other projects have zero NPV.

- If the firm invests in Project A the first year plus another project the second year that returns the opportunity cost of capital (10%), the total net present value is $90.91.
- If the firm invests in Project B the first year plus another project the second year that returns the opportunity cost of capital (10%), the total net present value is $181.82.

The IRR criterion implicitly assumes that other projects of the firm have the same opportunity cost of capital as the project being considered. This implies that the firm can invest in Projects A, B, or C each year.

- If the firm invests in Project A for two years in a row by re-investing the $11,100 in a similar project that returns 1.11 × $11,100 = $12,321.00 the second year, the total net present value is –$10,000 + $12,321 / 1.10^2 = $182.64.
- If the firm invests in Project B for two years in a row by re-investing the $11,200 in a similar project that returns 1.12 × $11,200 = $12,544.00 the second year, the total net present value is –$10,000 + $12,544 / 1.10^2 = $366.94.

Brealey and Myers assume that the net present value assumption is correct. When pricing insurance products, actuaries often assume that the internal rate of return assumption is correct. Both the SOA syllabus

(Atkinson and Dallas textbook on Course 5) and the CAS syllabus (Exam 9) assume the IRR assumption is correct.

The final exam will not ask why NPV is better than IRR (or vice versa). It may ask about the assumption in each criterion about the expected return of other projects.

Each assumption is appropriate in certain situations. As actuaries, you must recommend among various projects (that is, insurance products or insurance operations). You should understand the assumptions underlying each performance measurement criterion.

Exercise 4.8: Mutually Exclusive Projects

(Adapted from question 23 of the Spring 1997 Course 2 examination)

A firm has three mutually exclusive projects:.

	Project A	*Project B*	*Project C*
Initial Investment	$10,000	$15,000	$20,000
Net Cash Flow Year 1	$11,000	$11,000	$11,000
Net Cash Flow Year 2	$0	$6,000	$6,000
Net Cash Flow Year 3	$0	$0	$6,500

Assume that the opportunity cost of capital is 8%.

A. Which project has the largest net present value? Which project has the second largest?
B. What is the internal rate of return of Project A?
C. Divide Project B into Project A + Project (B – A). What are the internal rates of return of the two parts of Project B? Which project has the greater internal rate of return, Project A or Project B?
D. Divide Project C into Project B + Project (C – B) = Project A + Project (B – A) + Project (C – B). What are the internal rates of return of the three parts of Project C? Which project has the greatest internal rate of return, Project A, Project B, or Project C?
E. Why do the net present value and internal rate of return criteria rank these projects differently?

Solution 4.8:

Part A: The net present values are

- Project A: NPV = –$10,000 + $11,000 / 1.08 = $185.19.
- Project B: NPV = –$15,000 + $11,000 / 1.08 + $6,000 / 1.08^2 = $329.22.
- Project C: NPV = –$20,000 + $11,000 / 1.08 + $6,000 / 1.08^2 + $6,500/$1.08^3$ = $489.13.

Project C has the highest net present value, and Project B has the second highest.

Part B: We examine the incremental cash flows of the three projects to determine the internal rates of return.

Project A: –$10,000 + $11,000 / (1 + irr) = 0 ⇛ irr = $11,000 / $10,000 – 1 = 10.00%.

Part C: Project B – Project A = cash flows of –$5,000 at time 0 and +$6,000 at time 2. Brealey and Myers refer to this as the incremental cash flows of Project B over Project A. The internal rate of return of the incremental cash flows is ($6,000 / $5,000)^{½} –1 = 9.54%. Since 9.54% < 10%, Project B has a lower internal rate of return than Project A has.

Part D: The incremental cash flows between Projects B and C are –$5,000 at time 0 and +$6,500 at time 3. These cash flows have an internal rate of return of ($6,500 / $5,000)^{⅓} –1 = 9.14%. Since 9.14% < 10%, Project C has a lower internal rate of return than Project B has. Using the internal rate of return criterion, Project A is preferred to Project B which is preferred to Project C.

Part E: The NPV and IRR methods give different recommendations because they have different assumptions about unused cash. The NPV rule assumes that unused cash earns the opportunity cost of capital; the IRR rule assumes that unused cash earns the internal rate of return.

Project B uses $5,000 of cash that Project A does not use and earns a 9.54% return on that cash. The NPV rule assumes that if Project A used this cash, it would earn the 8% opportunity cost of capital, so Project B is preferred. The IRR rules assumes that if Project A used this cash, it would earn the same return that it earns on the cash which it does use. Since it earns 10% on the cash which it uses, it is preferred to Project B.

Brealey and Myers say the NPV assumption is correct and the IRR assumption is incorrect. We do not judge between the Brealey and Myers perspective and the common actuarial perspective; SOA Course 5 and CAS Exam 9 use the IRR perspective, which is more common in the insurance industry. The final exam will not ask you to judge between the two criteria, but it will ask you about the implicit assumptions in the two criteria.

Question: Suppose an insurer has $5 billion of surplus, its risk-based capital requirements with $10 billion of premium are $2 billion, and it has an A+ rating from Standard & Poor's and from A. M. Best. Its opportunity cost of capital is 10%. It can write $2 billion of premium at a 12% return, $2 billion at an 11% return, …, and $2 billion at an 8% return. From the theory in the text, the insurer should write only $6 billion of premium, since any additional writings earn a return less than the opportunity cost of capital. But why should the insurer forgo good business even if the return is less than 10%? It is not using the capital for other purposes, and its ratings even with $10 billion of business are excellent.

Answer: If there are no imperfections in the capital markets (such as taxes), the firm should write $6 billion of business and give back the unused capital to its shareholders, who can invest it at 10% per annum. In practice, many firms hate to return unneeded capital to their owners; even if they wanted to return the capital, federal income taxes may make it better to hold on the capital even if the best investment is less than the opportunity cost of capital.

Question 4.9: Book Rate of Return

(Adapted from question 3 of the Fall 1997 actuarial examination)

Project A requires an initial investment of $15,000, which will be depreciated over three years using straight line depreciation. The projected revenues and expenses are:

	Year 1	Year 2	Year 3
Revenue	12,000	8,000	8,000
Out-of-pocket Cost	7,000	2,000	2,000

What is the average annual book rate of return for the project?

{Note: The final exam for this course does not cover book rate of return. The CAS transition exam does cover book rate of return.}

Answer 4.9: We examine the net income and average book value each year:

	Year 1	*Year 2*	*Year 3*	*Average*
Revenue	12,000	8,000	8,000	
Out-of-pocket Cost	7,000	2,000	2,000	
Depreciation	5,000	5,000	5,000	
Net Income	0	1,000	1,000	666.7
Beginning Equity	15,000	10,000	5,000	
Ending Equity	10,000	5,000	0	

We have two ways to determine the average equity; they are equivalent when we use straight line depreciation. The average return on book value method is not a financial method, so neither version is necessarily better. Brealey and Myers use one version; most accountants use the other version. You will not be tested on the details of this method on the final exam, so you need not judge which version is correct. We do not venture an opinion on the best method of answer the CAS transition exam questions.

Brealey and Myers use the average equity of the four valuation dates starting with the beginning of the first year and ending with the end of the last year: this is

$$¼ \times (\$15,000 + \$10,000 + \$5,000 + \$0) = \$7,500.$$

Most accountants use the average of the average equity by year, or

$$⅓ \times (\$12,500 + \$7,500 + \$2,500) = \$7,500.$$

Since the depreciation here is pro-rata, the two methods give the same result.

The average income divided by the average book equity is $666.7 / $7,500 = 8.89%.

The final exam will not have numerical problems on the return on average book value method, since it is not a financial method of measuring performance. But the exam may ask basic questions about the method and its assumptions.

Question 4.10: Investment Rules

(Adapted from question 4 of the Fall 1997 actuarial examination)

An investment of $1,500,000 yields the following cash flows at the end of each year:

Year	*Net Cash Flow*
1	(200,000)
2	1,300,000
3	300,000
4	500,000

The opportunity cost of capital is 10% and the risk-free rate is 4%. Is this project worthwhile?

Answer 4.10: The project is not worthwhile. We determine the present value of each payment.

Year	*Net Cash Flow*	*Discounted Cash Flow*
1	(200,000)	(181,818)
2	1,300,000	1,074,380
3	300,000	225,394
4	500,000	341,507
Total		1,459,463

The net present value is the total discounted cash flows minus the initial investment of $1.5 million, or –$40,537 . This is negative, so the project should **not** be undertaken.

Exercise 4.11: Internal Rate of Return

(Adapted from question 25 of the Fall 1997 actuarial examination)

Rachel lends $1,000 to Jacob at the end of year 0. Jacob repays $300 at the end of year 1 and $900 at the end of year 2.

A. What is the internal rate of return of this loan? Is the internal rate of return the same for Jacob as for Rachel?
B. If the opportunity cost of capital is 10% per annum, who has the more favorable position, Jacob or Rachel?

Solution 4.11:

Part A: At the internal rate of return, the present value of cash inflows equals the present value of cash outflows: $1,000 = $300 / (1 + irr) + $900 / (1 + irr)^2. The cash outflows for Jacob are the inflows for Rachel, and vice versa; the internal rate of return is the same.

Letting (1 + irr) = z, we have

$$\$1{,}000 = \$300/z + \$900/z^2$$
$$10z^2 - 3z - 9 = 0$$
$$z = [\, 3 \pm (9 + 360)^{0.5} \,] / 20 \approx 1.110$$

Part B: Jacob borrows money; Rachel lends money. Jacob should borrow if the internal rate of return is *less* than the opportunity cost of capital; Rachel should lend if it is *greater* than the opportunity cost of capital. The internal rate of return of 11% exceeds the 10% opportunity cost of capital; so Rachel profits by lending money.

IRR vs NPV

Question: The difference between IRR and NPV is that NPV assumes unused cash earns the opportunity cost of capital and the IRR assumes unused cash earns the internal rate of return. Can you give an illustration to explain this?

Answer: Suppose an insurer has an opportunity cost of capital of 12% per annum. It has $1 million of capital, and it can sell life insurance or auto insurance in State W. This state does not allow an insurer to sell both life insurance and auto insurance.

The insurer's CEO asks an underwriter and an actuary to recommend either life insurance or auto insurance.

The underwriter compares a life insurance policy and an auto insurance policy.

~ The life insurance policy has a 10 year duration, a $100,000 initial investment, and an internal rate of return of 13%.
~ The auto insurance policy has a 1 year duration, a $20,000 initial investment, and an internal rate of return of 15%.

The underwriter reasons: "The opportunity cost of capital is 12% per annum and the insurance market is competitive. The average policy, whether life insurance or auto insurance, yields 12% per annum.

~ If we write life insurance, we get this policy plus other policies that yield 12%.
~ If we write auto insurance, we get this policy plus other policies that yield 12%.

The expected net present value of all other policies is zero, since they yield 12%, which is the opportunity cost of capital. This life insurance policy has a higher NPV than the auto insurance policy. The total NPV will be higher if we write life insurance.

The actuary compares the expected cash flows for life insurance and auto insurance.

~ Life insurance policies have 10 year durations, $100,000 initial investments, and internal rates of return of 13%.
~ Auto insurance policies have 1 year durations, $20,000 initial investments, and internal rates of return of 15%.

The actuary reasons: "I assume all the life insurance policies are the same and all the auto insurance policies are the same."

~ We can write 10 life insurance policies and hold each one for ten years. We make a 13% return on our investment of $1 million.
~ We can write 50 life insurance policies and renew each one ten times. We make a 15% return on our investment of $1 million.

Question: Who is correct?

Answer: If the market is perfectly competitive, all firms are identical, and the opportunity cost of capital is 12%, then the return on auto insurance can't be 15%. If it were 15%, other firms would enter the market and depress the return.

In practice, the returns may be higher or lower each year than the long-term average. The actuary above presumes the expected returns differ for life insurance vs auto insurance.