Corporate finance Mod 4: IRR practice problems

((The attached PDF file has better formatting.))

** Exercise 4.1: Size of Project

The opportunity cost of capital is 12% per annum.

An insurer sells term life insurance and Homeowners insurance in Florida. Each line of business requires capital of $50 million.

- The term life insurance has a profit of $10 million after one year.
- Because of bad hurricanes in the Gulf Coast, Homeowners rates have increased. The insurer expects to receive $12 million from Homeowners coverage after one year.

The insurer is not concerned about the risk of catastrophes since the risk is diversifiable. The profits include investment income from the original investment and all taxes.

The insurer is deciding whether to continue writing both lines of business or to write Homeowners alone.

- If the insurer writes both lines of business, it needs a $50 million initial investment for each line and expects to earn $10 million on term life and $12 million on Homeowners.
- If the insurer writes only Homeowners, it will write 50% more business, with 50% higher initial investment ($75 million) and a 50% higher profit after one year ($18 million).

A. What is the IRR for term life insurance?
B. What is the IRR for Homeowners?
C. What is the IRR for term life insurance and Homeowners if both are written?
D. What is the NPV for term life insurance?
E. What is the NPV for Homeowners if it is written with term life insurance?
F. What is the NPV for Homeowners if it is written alone?
G. What is the NPV for term life insurance and Homeowners if both are written?

Part A: The IRR for term life is $10 million / $50 million = 20.00%.

Alternatively: –$50 million + $60 million / (1 + IRR) = 0 ⇛ IRR = $60 million / $50 million – 1 = 20.00%.

Part B: The IRR for Homeowners is $12 million / $50 million = 24.00%.

Alternatively: –$50 million + $62 million / (1 + IRR) = 0 ⇛ IRR = $62 million / $50 million – 1 = 24.00%.

Writing 50% more Homeowners doesn't change the IRR, which remains 24%.

Part C: The IRR for term life and Homeowners combined is $22 million / $100 million = 22.00%.

Alternatively: –$100 million + $22 million / (1 + IRR) = 0 ⇛ IRR = $122 million / $100 million – 1 = 22%.

Homeowners written alone has the highest IRR.

Part D: The NPV for term life is $60 million / 1.12 – $50 million = $3.57 million.

Part E: The NPV for Homeowners (if it is written with term life) is $62 million / 1.12 – $50 = $5.36 million.

Part F: The NPV for term life plus Homeowners is $3.57 million + $5.36 million = $8.93 million.

Part G: The NPV for Homeowners written alone is ($62 million / 1.12 – $50) × 1.5 = $8.04 million.

The NPV for term life plus Homeowners is higher than for Homeowners written alone.

If two projects have positive net present values but different IRR's, the combination of the two projects has a higher NPV and a weighted average IRR.

** Exercise 4.2: Internal Rate of Return

A project has an initial cash outflow followed by cash inflows with a 12% *per annum* opportunity cost of capital.

Explain whether each of the following is true?

A. IRR does not consider the time value of money.
B. The IRR criterion should not be used to choose among mutually exclusive projects of different size.
C. If the project has a *positive* NPV, its IRR exceeds 12%.
D. If the IRR indicates that a loan is desirable for the lender, it indicates that it is desirable for the borrower.
E. If the IRR of the project is less than 12%, it has a negative NPV.

Part A: False: IRR considers the time value of money, since it solves for the interest rate that sets the present value of the cash flows to zero.

Part B: True: The IRR criterion should be used to choose among mutually exclusive projects of the same size and duration. If projects have different sizes or durations, Brealey and Myers say to use NPV, not IRR.

Parts C and E: True. The relation of NPV and IRR is

NPV	IRR
positive	IRR > opportunity cost of capital
negative	IRR < opportunity cost of capital
zero	IRR = opportunity cost of capital

Part D: False. For a loan, the cash inflow precedes the cash outflows for the borrower; the opposite is true for the lender. The borrow prefers a loan with a low IRR; The lender prefers a loan with a high IRR.

Question: People profit from projects with positive NPV's. If the borrower and lender have opposite NPV's for a loan, why are they made?

Answer: A loan is made when the borrower and lender have different opportunity costs of capital.

Illustration: A consumer wants to buy an expensive item, such as a home or a car, and applies for a bank loan.

- The bank's opportunity cost of capital may be 8% per annum, since it can borrow funds by issuing bonds.
- The consumer next best source of funds may charge 12% per annum.

A loan at 10% per annum is a positive NPV project for both the bank and the consumer.

** Exercise 4.3: NPV and IRR

Projects Q, R, and S each have one cash outflow at time t=0 (the initial investment), and one cash inflow at either time t=1 or time t=2 (one year vs two year duration). The opportunity cost of capital is 12% per annum.

- Project Q has a one year duration and an initial investment of $9,000.
- Project R has a one year duration and an initial investment of $19,000.
- Project S has a two year duration and an initial investment of $19,000.

All three projects have a net present value of $1,000.

A. What are the cash inflows at the end of each project?
B. What are the IRR's for each project?

Part A: Work out the return at the expiration of the project based on the NPV of $1,000 and the capitalization rate of 12%. Add the NPV to the initial investment and multiply by the opportunity cost of capital for the period until the end of the project to get the cash inflow at time t=1 or time t=2. Then work out the IRR of the project as the return which gives a present value of zero.

$$-CF_0 + CF_N / (1.12)^N = \$1{,}000 \Rrightarrow CF_N = (\$1{,}000 + CF_0) \times 1.12^N$$

- *Project Q:* ($1,000 + $9,000) × 1.12 = $11,200
- *Project R:* ($1,000 + $19,000) × 1.12 = $22,400
- *Project S:* ($1,000 + $19,000) × 1.12^2 = $25,088

Part B: The IRR of the project is the return which gives a present value of zero.

$$-CF_0 + CF_N / (1 + IRR)^N = 0 \Rrightarrow (CF_N / CF_0)^{1/N} - 1$$

This cash inflow divided by the initial investment, raised to the power of the reciprocal of the duration of the project, is the IRR:

- *Project Q:* 10,000 × 1.12 / 9,000 = 1.244
- *Project R:* 20,000 × 1.12 / 19,000 = 1.179
- *Project S:* $[20{,}000 \times 1.12^2 / 19{,}000]^{½}$ = 1.149

Project Q has the highest IRR and Project S has the lowest IRR.

** Exercise 4.4: NPV and IRR

Projects Q, R, and S each have one cash outflow at time t=0 (the initial investment), and one cash inflow at either time t=1 or time t=2 (one year vs two year duration). The opportunity cost of capital is 12% per annum.

- Project Q has a one year duration and an initial investment of $10,000.
- Project R has a one year duration and an initial investment of $20,000.
- Project S has a two year duration and an initial investment of $20,000.

All three projects have an internal rate of return of 13%.

A. What are the cash inflows for each project?
B. What the net present values for each project?

Part A: Work out the cash inflows at the end of the project:

- *Project Q:* The cash flow in one year is $10,000 × 1.13 = $11,300.
- *Project R:* The cash flow in one year is $20,000 × 1.13 = $22,600.
- *Project S:* The cash flow in two years is $20,000 × 1.13^2 = $25,538.

Part B: Work out the net present values for each project.

- *Project Q:* The net present value is –$10,000 + $11,300 / 1.12 = $89.29
- *Project R:* The net present value is –$20,000 + $22,600 / 1.12 = $178.57
- *Project S:* The net present value is –$20,000 + $25,538 / 1.12^2 = $358.74

Intuition: Project Q has a positive NPV, since the internal rate of return is more than the opportunity cost of capital.

- Project R is two copies of Project Q side by side, so its NPV is twice as great.
- Project Q is two copies of Project R one after another, so its NPV is about twice as great.

Two copies of Project R side by side are 2 × $178.57 = $357.14. Since the second project is 13% greater, not 12% greater, we increase the second project by 1.13 / 1.12:

$$\$178.57 + \$178.57 \times 1.13 / 1.12 = \$358.73$$