## Corporate Finance, Module 6, "Risk, Return, and the Opportunity Cost of Capital"

## Geometric Average vs Arithmetic Average

(The attached PDF file has better formatting.)

Illustration: Geometric vs Arithmetic Average
An investment returns either $0 \%$ or $20 \%$ each year, with a $50 \%$ probability of each. What is the expected annual return?

Question: Consider the expected return over two years. The investment may earn 0\% the first year and 20\% the next year, or $20 \%$ the first year and $0 \%$ the next year. Either way, the two year return is $20 \%$, which is equivalent to $1.20^{1 / 2}-1=9.54 \%$ per annum. We use the geometric average, not the arithmetic average.

Answer: You are correct that 20\% over two years is equivalent to $9.54 \%$ over one year. But the expected return over two years is not $20 \%$. The investment has four possible returns, not two:

|  | Returns |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Scenario | Year 1 | Year 2 | Two Year | Probability |
| 1 | $0 \%$ | $0 \%$ | $0 \%$ | $25 \%$ |
| 2 | $0 \%$ | $20 \%$ | $20 \%$ | $25 \%$ |
| 3 | $20 \%$ | $0 \%$ | $20 \%$ | $25 \%$ |
| 4 | $20 \%$ | $20 \%$ | $44 \%$ | $25 \%$ |
| Total |  |  | $84 \%$ | $100 \%$ |

The expected return is $84 \% / 4=21 \%$. The expected annual return is $1.21^{1 / 2}-1=10 \%$. This is the arithmetic average of $0 \%$ and $20 \%$.

