Corporate Finance, Module 7: “Risk and Return” (Chapter 8)

Homework Assignments

(The attached PDF file has better formatting.)

The first two homework assignments cover the basics of the CAPM. The third homework assignment shows how to combine portfolios to achieve different risk-return combinations. If the CAPM is correct, all combinations must lie on the same risk-return line, and the differences in the third homework assignment would not be observed.

Exercise 7.1: Betas and Expected Returns

Suppose the risk-free interest rate is 7% per annum and the expected return on the overall market is 15% per annum. Assume the CAPM holds.

A. What is the market risk premium? { This is $E(r_m) – r_f$. }
B. If a stock has a beta of 1.25, what is the required return on that stock? (The stock’s required return is the risk-free rate plus its beta times the market risk premium.)
C. If the expected return of a stock is 11%, what is its beta? (We invert the CAPM formula to solve for the beta in terms of the required return.)

(Later modules differentiate between the beta of a stock’s equity and the beta of a stock’s assets; this homework assignment deals with the beta of the stock’s equity.)

(Brealey and Myers are advocates of the CAPM, and the NEAS on-line course follows their view. Many final exam problems use the CAPM, both for mathematical problems and for conceptual questions.)

Exercise 7.2: Risk-free Rate and Market Risk Premium

Suppose we have the following information about two stocks:

	Expected Return	*Beta*
Stock #1	16%	0.8
Stock #2	23%	1.5

If the CAPM holds, what are the risk-free interest rate and the market risk premium? (You must solve a pair of linear equations. Since one unknown, the risk-free rate, has a coefficient of one in both equations, the market risk premium times the difference in betas is the difference in the expected return.)

Exercise 7.3: Combining Portfolios

An investor has a choice of three portfolios with the following expected returns and betas:

Portfolio	*Expected Return*	*Beta*
Portfolio #1	12%	0.5
Portfolio #2	16%	1.1
Portfolio #3	20%	2.0

A. If the investor combines portfolios #1 and #2 to give the same expected return as portfolio #3, what is the beta of the combined portfolio? (Solve: $\alpha \times 12\% + (1 - \alpha) \times 16\% = 20\%$; α is negative, meaning that the investor sells portfolio #1 and buy more than 100% of portfolio #2. We refer to this as a short position in one portfolio and a long position in the other portfolio.) Is it higher or lower than the beta of portfolio #3? (In practice, it should not be higher or lower than the return on portfolio #3, since arbitragers would force the returns to be equal. If we solved for the risk-free rate and the market risk premium from portfolios #1 and #2, there values would not give the expected return in portfolio #3.)
B. If the investor combines portfolios #1 and #2 to have the same beta as portfolio #3, what is the expected return of the combined portfolio? Is it higher or lower than the expected return of portfolio #3? (Solve: $\alpha \times 0.5 + (1 - \alpha) \times 1.1 = 2.0$)
C. If the investor combines portfolios #1 and #3 to give the same expected return as portfolio #2, what is the beta of the combined portfolio? Is it higher or lower than the beta of portfolio #2?
D. If the investor combines portfolios #1 and #3 to have the same beta as portfolio #2, what is the expected return of the combined portfolio? Is it higher or lower than the expected return of portfolio #2?
E. If the investor combines portfolios #2 and #3 to give the same expected return as portfolio #1, what is the beta of the combined portfolio? Is it higher or lower than the beta of portfolio #1?
F. If the investor combines portfolios #2 and #3 to have the same beta as portfolio #1, what is the expected return of the combined portfolio? Is it higher or lower than the expected return of portfolio #1?
G. Based on the answers to Parts A-F, which portfolio seems best and which portfolio seems worst? (The best portfolio has a higher return for a given beta than the other two portfolios provide; the worst portfolio has a lower return.)

(Several of the combined portfolios require a *short* position in one portfolio and a *long* position in the other portfolio.)

Question: It seems that a more risk averse investor might choose Portfolio #1 and a less risk averse investor might choose Portfolio #3. How can we say that one portfolio is better than another?

Answer: Suppose an investor has a choice of two portfolios with the same market value:

- Portfolio Y has a beta of 1.0 and yields 12% per annum.
- Portfolio Z has a beta of 1.0 and yields 16% per annum.

An investor would buy Portfolio Z and sell short Portfolio Y. The combined portfolio has a β of zero and yields 4% per annum. The investor has a practically risk-free portfolio that cost no money and yields a positive income.

Question: Isn't this what hedge funds do? Hedge funds buy (long) and sell (short) portfolios with slight differences to bet on movements in sector returns (or interest rates or any other item). They might earn large profits or large losses without putting up much capital. This is very risky. Here also, Portfolios Y and Z have stochastic returns. They might increase or decrease in value from random fluctuations.

Answer: Portfolios Y and Z differ in their unique risk; their systematic risk is identical. The unique risk (non-systematic risk) can be eliminated by diversification. If the total portfolio is well diversified, this part of the portfolio has zero risk and a positive return.

Question: This exercise doesn't have two stocks with the same beta and different returns.

Answer: For each stock, construct matching portfolios.

- One matching portfolio has the same beta but a higher or lower expected return.
- The other matching portfolio has the same expected return but a higher or lower beta.

Question: How do we know which portfolio is best?

Answer: Suppose we form portfolios with matching betas and higher or lower expected returns. We get one of two scenarios:

- One portfolio is better than its matching portfolio; the other two are worse than their matching portfolios. The first portfolio is the best.
- One portfolio is worse than its matching portfolio; the other two are better than their matching portfolios. The first portfolio is the worst.

Question: We infer either the best or the worst portfolio. Can we rank all three?

Answer: Consider the first scenario. Suppose two portfolios are worse than their matching portfolios. The expected returns may 1% too low for one portfolio and 3% too low for the other portfolio. We can't always rank the portfolios, but often we can.