Corpfin mod 7: Risk and return - intuition and practice problems

(Brealey and Myers Chapter 8 CAPM)

(The attached PDF file has better formatting.)

** Exercise 7.1: Standard deviation and expected return

Suppose the market has two stocks.

- Stock ABC has an expected return of 12% and a standard deviation of 25%.
- Stock XYZ has an expected return of 15% and a standard deviation of 30%.
- The correlation of the returns on the two stocks is 40%.

A. What is the minimum standard deviation for any portfolio of these two stocks?
B. What is the expected return for this minimum standard deviation portfolio?

Part A: If portfolio has α of Stock ABC and $(1 - \alpha)$ of Stock XYZ, the variance of the portfolio is

$0.25^2 \times \alpha^2 + (1 - \alpha)^2 \times 0.30^2 + 2\ \alpha\ (1 - \alpha)\ 40\% \times 0.25 \times 0.30$

Set the partial derivative with respect to α equal to zero to get:

$2 \times 0.25^2 \times \alpha + (-1) \times 2 \times (1 - \alpha) \times 0.30^2 + 2 \times 40\% \times 0.25 \times 0.30 \times (1 - 2\alpha) = 0$

$0.25^2 \times \alpha - (1 - \alpha) \times 0.30^2 + 40\% \times 0.25 \times 0.30 \times (1 - 2\alpha) = 0$

$\alpha \times (0.25^2 + 0.30^2 - 2 \times 40\% \times 0.25 \times 0.30) = 0.30^2 - 0.40 \times 0.25 \times 0.30$

$\alpha = (0.30^2 - 0.40 \times 0.25 \times 0.30) / (0.25^2 + 0.30^2 - 2 \times 40\% \times 0.25 \times 0.30) = 64.8649\%$.

The variance is

$0.25^2 \times 0.648649^2 + (1 - 0.648649)^2 \times 0.30^2 + 2 \times 0.648649 \times (1 - 0.648649) \times 40\% \times 0.25 \times 0.30 = 0.05108$

The standard deviation is $0.05108^{0.5} = 0.22601 = 22.6\%$.

Part B: The expected return is $0.648649 \times 12\% + (1 - 0.648649) \times 15\% = 13.05\%$

Note: The Brealey and Myers textbook is written for college students with little mathematics background. It does not show how to maximize or minimize the standard deviation or expected return by taking derivatives. The final exam problems assume you know how to maximize or minimize a mathematical expression.

** Exercise 7.2: Efficient frontier

Five stocks are trading with the following expected returns and standard deviations.

	A	B	C	D	E
Expected Return	12.5%	15.0%	17.0%	18.0%	20.0%
Standard Deviation	21.0%	25.0%	29.5%	31.0%	45.0%

Investors can hold any combination of these stocks, such as 50% of Stock A and 50% of Stock B.

A. Which stock is not on the efficient frontier?
B. If investors can borrow and lend at the risk-free rate of 12%, which of the stocks has the highest Sharpe ratio?

Part A: Instead of Stock C, an investor can form a portfolio with one third Stock B and two thirds Stock D. This portfolio has an expected return of $\frac{1}{3} \times 15\% + \frac{2}{3} \times 18\% = 17\%$. Its maximum possible standard deviation (with a perfect partial correlation of $\rho = 100\%$ of the stock returns) is $\frac{1}{3} \times 25\% + \frac{2}{3} \times 31\% = 29\%$. This has the same expected return as Stock C and a lower standard deviation.

Question: Do individual stocks lie on the efficient frontier?

Answer: The efficient frontier has (mostly) diversified portfolios of stocks. A stock which has no unique risk (over systematic risk) might lie on the efficient frontier, and the stock with the highest expected return also lies on the efficient frontier. Most other stocks lie below the efficient frontier.

Question: Isn't there only one diversified portfolio, which is the market portfolio?

Answer: There are millions of diversified portfolios. Any portfolio of 100 or so unrelated stock is diversified; the non-systematic risk is not material. One can form diversified portfolios with almost any beta, since the market has so many stocks.

Part B: Form Sharpe ratios for each stock:

	A	B	C	D	E
Expected Return	12.5%	15.0%	17.0%	18.0%	20.0%
Standard Deviation	21.0%	25.0%	29.5%	31.0%	45.0%
Sharpe Ratio	2.38%	12.00%	16.95%	19.35%	17.78%

Stock D has the highest Sharpe ratio (19.35%).

** Exercise 7.3: Expected returns

- Stock ABC has a CAPM beta of 0.800.
- Stock XYZ has a CAPM beta of 1.200.

The market risk premium is twice the annual risk-free rate.

What is the ratio of XYZ's expected return to ABC's expected return?

Solution 7.3:

- The expected return for Stock ABC is $r_f + 0.8 \times 2 \times rf = 2.6 \times r_f$, where r_f is the risk-free rate.
- The expected return for Stock XYZ is $r_f + 1.2 \times 2 \times rf = 3.4 \times r_f$, where r_f is the risk-free rate.

The ratio of XYZ's expected return to ABC's expected return is 3.4 / 2.6 = 1.30769.

** Exercise 7.4: Expected returns and stock prices

In 20X1, the risk-free rate is 5% and the market risk premium is 8%.

- In 20X2, personal income taxes are repealed.
- People feel wealthier and become less risk averse.
- The expected cash flows from stocks do not change.

A. Do people want to hold more or less stocks in their portfolios?
B. Does the market risk premium increase, decrease, or stay the same?
C. Do stock prices increase, decrease, or stay the same?

Part A: The market risk premium reflects risk aversion of investers. Higher risk aversion causes a higher market risk premium, since risk averse investor prefers bonds to stocks. If people become less risk averse, they are more likely than before to buy stocks.

Part B: Investors need less excess return to prefer stocks, so the market risk premium decreases.

Part C: The future cash flows from the stock remain the same. The market risk premium decreases, so the capitalization rate decreases, and the stock price increases.

Question: Could you give an example of this?

Answer: In the 1990's, stocks had a long and powerful rally. Some economists presumed that lower tax rates and improved business conditions from the Reagan years, the decline of the Soviet threat to U.S. security and the hoped-for "peace dividend," and the computer advances (world wide wed, personal computers) led to greater consumer wealth and lower risk aversion.

** Exercise 7.5: Security market line

The security market line is plotted on a graph, rising from the lower left to the upper right.

A. What is the horizontal axis of the graph?
B. What is the vertical axis of the graph?
C. Where on the graph do undiversified stock portfolios lie?
D. If a stock lies below the security market line, would demand for the stock be high or low?
E. How would competitive pressures move the stock's position on the graph?
F. If a stock lies above the security market line, would demand for the stock be high or low?
G. How would competitive pressures move the stock's position on the graph?

Part A: The horizontal axis is the CAPM beta of the stock.

Part B: The vertical axis is the expected return of the stock.

Part C: All stocks lie on the security market line. Stock portfolio expected returns and CAPM betas are weighted averages of the expected returns and CAPM betas of the individual stocks, so all portfolios also lie on the security market line.

Part D: A stock lying below the security market line has a high beta for its expected return, or a low expected return for its beta. Brealey and Myers assumes the price elasticity of demand for stocks is infinite: that is, the absolute value of the elasticity is very high, so demand for the stock is close to zero.

Part E: If demand is close to zero, the stock's price would decrease, and its expected return would increase. The stock would move vertically upward.

Part F: A stock lying above the security market line has a low beta for its expected return, or a high expected return for its beta. Brealey and Myers assumes the price elasticity of demand for stocks is infinite: that is, the absolute value of the elasticity is very high, so demand for the stock is extremely high. All investors want to buy the stock; no one wants to sell it.

Part G: If demand is extremely high, the stock's price would increase, and its expected return would decrease. The stock would move vertically downward.

** Exercise 7.6: Efficient frontier

The efficient frontier is plotted on a graph.

A. What is the horizontal axis of the graph?
B. What is the vertical axis of the graph?
C. Is the efficient frontier upward sloping or downward sloping?
D. Where on the graph do fully diversified stock portfolios lie?
E. Where on the graph do undiversified stock portfolios lie?
F. If a stock lies below the efficient frontier, would demand for the stock be high or low?

Part A: The horizontal axis is the standard deviation of the stock.

Part B: The vertical axis is the expected return of the stock.

Part C: The efficient frontier is upward sloping.

Part D: The efficient frontier is portfolios with the minimum standard deviation for a given expected return. Diversifying a stock portfolio eliminates non-systematic risk. Systematic risk can not be eliminated, so this portfolio lies on the efficient frontier.

Part E: Consider two portfolios with expected return of Z. Portfolio ABC is fully diversified; portfolio XYZ is not fully diversified. ABC and XYZ have the same systematic risk, since they have the same expected return. ABC's standard deviation is only its systematic risk; XYZ's standard deviation reflects both systematic risk and other risk. XYZ has the higher standard deviation, so it lies to the right of ABC. Since ABC lies on the efficient frontier, XYZ lies below the efficient frontier.

Part F: A stock that lies below the efficient frontier has at least some non-systematic risk. Demand for the stock depends on its CAPM beta and its expected return. All stocks has some non-systematic risk, so all stocks lies below the efficient frontier. The stock market is very efficient, so stock prices change quickly to offset changes in demand. All stocks have similar demand, regardless where they lie on the graph.

** Exercise 7.7: Optimal portfolios

- The risk-free rate is 7% per annum.
  - Investors can borrow and lend at this rate.
- All investors hold a combination of risk-free assets and the market portfolio.

- Investor ABC puts half his money in risk-free assets and half in the market portfolio.
  - The expected return of the portfolio is 11% and the standard deviation is 15%.
- Investor XYZ borrows an amount equal to his own funds and invests all of it in the market portfolio.

A. What is the market risk premium?
B. What is the expected return from XYZ's portfolio?
C. What is the standard deviation of returns from XYZ's portfolio?

Part A: The expected return for ABC's portfolio is $\frac{1}{2} \times (r_f + r_m)$.

The expected excess return for ABC's portfolio is $11\% - 7\% = [\frac{1}{2} \times (r_f + r_m) - r_f\,] = \frac{1}{2} \times (r_m - r_f) = \frac{1}{2} \times$ the market risk premium $\Rightarrow$ the market risk premium $= 2 \times (11\% - 7\%) = 8\%$.

Part B: XYZ buys twice as much of the market portfolio as his funds and pays interest on this funds.

The expected return for XYZ's portfolio is $2r_m - r_f = 2 \times (7\% + 8\%) - 7\% = 23\%$.

Part C: The standard deviation of the risk-free assets is zero. The standard deviation of the market portfolio is 30%, so the standard deviation of XYZ's portfolio is $2 \times 30\% = 60\%$.

** Exercise 7.8: Optimal portfolios

- Investors can borrow and lend at the risk-free rate r_f
- All investors hold a combination of risk-free assets and the market portfolio, which has an expected return of r_m and a standard deviation of *sd*.

- Investor ABC puts half his money in risk-free assets and half in the market portfolio.
- Investor XYZ borrows an amount equal to his own funds and invests all of it in the market portfolio.

A. What is the expected return for ABC's portfolio?
B. What is the standard deviation for ABC's portfolio?
C. What is the Sharpe ratio for ABC's portfolio?
D. What is the expected return for XYZ's portfolio?
E. What is the standard deviation for XYZ's portfolio?
F. What is the Sharpe ratio for XYZ's portfolio?

Part A: The expected return for ABC's portfolio is $\frac{1}{2} \times (r_f + r_m)$.

Part B: The standard deviation for ABC's portfolio is $\frac{1}{2} \times sd$.

Part C: The Sharpe ratio for ABC's portfolio is $[\frac{1}{2} \times (r_f + r_m) - r_f\,] / (\frac{1}{2} \times sd) = (r_m - r_f) / sd$

Part A: The expected return for XYZ's portfolio is $2r_m - r_f$.

Part B: The standard deviation for ABC's portfolio is $2 \times sd$.

Part C: The Sharpe ratio for ABC's portfolio is $[2r_m - r_f. - r_f\,] / (2 \times sd) = (r_m - r_f) / sd$

Question: The Sharpe ratios are the same for all investors, regardless of the portion of the portfolio held in risky vs risk-free assets. It seems that the Sharpe ratio is a useless measure of risk.

Answer: The Sharpe ratios differ for each risky portfolio. The market portfolio has the highest Sharpe ratio. Mixing the market portfolio with risk-free assets does not change the Sharpe ratio.

** Exercise 7.9: Market anomalies

Economists use empirical data to test the validity of the CAPM. All stocks are placed in 10 groups, depending on their betas. Group 1 has the 10% of stocks with the lowest betas; Group 2 has the 10% of stocks with the next lowest betas; …; Group 10 has the 10% of stocks with the highest betas.

Each portfolio (group) is examined over many years. The portfolio returns are graphed.

- The horizontal axis is the portfolio's beta.
- The vertical axis is the portfolio's annual return minus the risk-free rate.

Question: The risk-free rate differs each year. What risk-free rate do we use?

Answer: Each year the portfolio's excess return is its actual return minus the risk-free rate in that year. The vertical axis is the average excess return.

Question: Stock betas change; they are not stable. Suppose a stock has a low beta the first year and a high beta the last year (or vice versa). In which portfolio is the stock placed?

Answer: The portfolios are re-formed each year. Group 1 is investors who hold the 10% lowest beta stocks that year. Brealey and Myers do not discuss these details. It is simpler to imagine the study as if stock betas do not change from year to year.

A. If the CAPM is correct, what is the shape of the graph: a straight line, concave, or convex?
B. What is the actual shape of the graph?
C. If we draw a regression line through the ten points, is the slope of the regression line higher or lower than the slope of the security market line?
D. Which portfolios have high risk adjusted returns? Which portfolios have low risk adjusted returns?

Part A: The axes of this graph are those of the security market line. If the CAPM is correct, all stocks lie on the security market line, so all stock portfolios lie on the security market line. The graph is an upward sloping straight line. The line passes through the origin: if the beta of the portfolio is zero, the excess return is zero. The slope of the line is the market risk premium: if beta (horizontal axis) increases by one, the excess return (vertical axis) increases by the market risk premium.

Part B: The graph actually has a S-shape.

- Portfolios with betas near one lie on the security market line.
- Portfolios with very low betas lie above the security market line.
- Portfolios with very high betas lie below the security market line.

See the three graphs on page 197 (Chapter 8 of the tenth edition of Brealey and Myers). Figure 8.8 shows the S-curve most clearly. The bottom graph (1966-2008) is unusual in that high beta stocks perform worse than medium beta stocks.

Part C: The regression line is flatter than the security market line (it has a lower slope). But it is not a straight line with a lower slope, which would indicate that the CAPM is correct but the market risk premium is too high. Rather it is an S-shaped curve. The market risk premium of the CAPM is correct for stocks with betas close to one. It is too high for high beta stocks and low beta stocks.

Part D: Low beta stocks have high risk adjusted returns. High beta stocks have low risk adjusted returns. Know the comments in Figure 8.8: "The high beta portfolios generated higher average returns, just as predicted by the CAPM. But the high beta portfolios plotted below the market line, and the low beta portfolios plotted above. A line fitted to the ten portfolios would be 'flatter' than the market line."

** Exercise 7.10: Market anomalies

We compare actual returns of small firm ("small cap") stocks vs large firm ("large cap") stocks and of value stocks vs growth stocks. Value stocks have high ratios of book value to market value; growth stocks have low ratios of book value to market value. "Small cap" means small capitalization.

A. If the CAPM is correct, and each group of stocks has an average beta of one, what would one expect from these comparisons?
B. What is the actual result of these comparisons?
C. If the CAPM is correct, what should happen when these results are published?
D. How might data mining explain these results?

Part A: If the CAPM is correct, the only influence of expected returns is the covariance with the market returns. Small firms should have the same risk adjusted returns as large firms, and value stocks should have the same risk adjusted returns as growth stocks.

Part B: Brealey and Myers (page 198 of Chapter 8 of the tenth edition) show that "over the long haul, owners of small firm stocks would have made substantially higher returns" and that "value stocks have provided a higher long-run return than growth stocks."

Part C: Brealey and Myers say (page 198 of Chapter 8 of the tenth edition): "Maybe the size and book-to-market effects are simply chance results that stem from data-snooping. If so, they should have vanished once they were discovered. There is some evidence that this is the case."

Part D: Perhaps only CAPM betas influence expected returns, but common stocks returns have much random fluctuation. We test various items with a 5% significance level, meaning that a result is believed only if it would occur 5% or less of the time from random fluctuations alone. If we test 20 items, one of them (on average) would be significant, even if all have no true influence on common stock returns. Economists have tested dozens of potential influences on common stock returns; a few of them show significant effects, even though all may be irrelevant.

Question: Brealey and Myers explain away these market anomalies. Do the empirical data validate the CAPM?

Answer: Brealey and Myers are strong believers in the CAPM. The empirical data in these studies (by Fama and French in 1992 and 1993, and replicated several times afterward) shows that the CAPM betas have almost no predictive power for future returns. It is not just that the CAPM is an incomplete predictor of future returns; it is not a good predictor at all. Data mining is not a valid explanation. Income vs growth stocks (that is, the book to market ratio) has a strong effect, with a significance level that is too strict for data mining.

** Exercise 7.11: Efficient frontier

The efficient frontier is plotted on a graph.

A. Is the efficient frontier concave or convex?
B. Suppose two portfolios ABC and XYZ lie on the efficient frontier. A third portfolio is formed as DEF = ½ × (ABC + XYZ). Does DEF lie above, below, or on the efficient frontier?

Part A: The efficient frontier is concave. To see this, suppose two portfolios ABC and XYZ lie on the efficient frontier, and the efficient frontier is convex between ABC and XYZ. A third portfolio is formed as DEF = ½ × (ABC + XYZ). The expected return of DEF is the straight average of the expected returns of ABC and XYZ. The standard deviation of DEF is less than or equal to the straight average of the standard deviations of ABC and XYZ. DEF must lie to the left of the efficient frontier. But no portfolio can lie to the left of the efficient frontier. By definition, the efficient frontier is the portfolio with the minimum standard deviation for a given expected return.

Part B: From the reasoning in Part A, portfolio DEF lies below or on the efficient frontier.