Corporate Finance, Module 11: Maximizing NPV practice problems

(The attached PDF file has better formatting.)

** Exercise 11.1: Economic vs Accounting Income and Depreciation

A firm invests $10,000 in machinery that yields net cash flows of $7,000 at the end of each of the next two years. For accounting purposes, the machinery is depreciated pro-rata over two years, with no salvage value. The opportunity cost of capital is 10%.

A. What is the accounting depreciation in years 1 and 2?
B. What is the accounting income in years 1 and 2?
C. What is the economic depreciation in years 1 and 2?
D. What is the economic income in years 1 and 2?
E. What is the net present value of the project?
F. If the machinery costs $12,000 instead of $10,000, how do Parts A through E change?

Part A: Pro-rata depreciation means equal dollar amounts in each year; the total depreciation is the purchase price minus the salvage value. Pro-rata depreciation over two years is $10,000 / 2 = $5,000 each year.

Part B: Accounting income is the cash flow minus depreciation, or $7,000 – $5,000 = $2,000 each year.

Part C: Economic depreciation is the decrease over the year in the present value of future cash flows. These present values are

End of Year	*PV(Future Cash Flows)*
0	$7,000 / 1.10 + $7,000 / 1.10^2 = $12,148.76
1	$7,000 / 1.10 = $6,363,64
2	0

- The economic depreciation in year 1 is $12,148.76 – $6,363.64 = $5,785.12.
- The economic depreciation in year 2 is $6,363.64 – 0 = $6,363.64.

Part D: Economic income is the cash flow minus the economic depreciation:

- The economic income in year 1 is $7,000 – $5,785.12 = $1,214.88
- The economic income in year 2 is $7,000 – $6,363.64 = $636.36

Question: Is there a more straight-forward way to think of economic income?

Answer: We compare GAAP income and economic income:

- GAAP income / beginning of the year GAAP equity = return on book equity.
- Economic income / beginning of the year economic value = capitalization rate.

The expected economic income (in any year) divided by the project's present value at the beginning of the year is the capitalization rate for the project. Reverse the formula to say:

present value at end of the previous year × the capitalization rate = expected economic income for the year.

- The present value at the end of year 0 is $12,148.76 ⇒ economic income in year 1 is $12,148.76 × 10% = $1,214.88.

- The present value at the end of year 1 is \$6,363.64 ⇒ economic income in year 2 is \$6,363.64 × 10% = \$636.36.

Question: Why do Brealey and Myers define economic income as cash flow minus economic depreciation?

Answer: Accounting income is cash flow minus accounting depreciation. They extend the formula to economic income. Also, the present value at the end of the previous year × the capitalization rate gives the *expected* economic income. If the actual cash flows differ from the expected cash flows, actual economic income differs from expected economic income.

Question: In this exercise, the accounting figures balance. The cash flow is the same each year, so depreciation and income are the same each year. Why is this not true for economic depreciation and economic income?

Answer: The accounting income statement has the same cash flows, accounting depreciation, and accounting income. The return on book equity differs from year to year: it is lowest at inception of the project, when book equity is highest, and highest at the end of the project, when book equity is lowest. This has two problems:

- Returns on book equity may range widely for long-term projects.
- If future cash flows are correctly estimated, the profit or loss should be allocated to the period when the business decision is made to accept the project. GAAP allocates no profit to the moment when the business decision is made; all income is deferred over the term of the project. In contrast, the net present value is allocated entirely to the period when the business decision is made, whether it is a profit or a loss.

Economic income is not the measure of profit or loss; the measure is the net present value. Economic income shows the unwinding of the discount over the term of the project. If the project's systematic risk does not change, the capitalization rate does not change. The capitalization rate is the item that stays constant in economic accounting.

Part E: The net present value of the project is \$12,148.76 – \$10,000 = \$2,148.76.

Part F: If the machinery costs \$12,000 instead of \$10,000, the accounting depreciation is \$6,000 each year and the accounting income is \$1,000 each year.

The calculations of economic depreciation and economic income worked out above *do not consider the cost of the machinery*, so there are no changes to economic depreciation and economic income.

Question: If the machinery costs more, the project is less profitable. Shouldn't depreciation be higher and income lower?

Answer: If the machinery costs \$12,000 instead of \$10,000, the net present value of the project is \$148.76 instead of \$2,148.76. The *net present value* of the project (at inception) declines; the income in subsequent years does not change. Changing the purchase price of the machine does not change the present value of *future* cash flows, so it does not change the economic worth of the project. Since the capitalization rate does not change, the economic income does not change, so the economic depreciation does not change.

Question: Suppose a project has some depreciable assets, such as buildings or machinery, and some non-depreciable assets, such as land and technological know-how. Accounting depreciation applies only to depreciable assets, and applies over the estimated useful life of the asset. Economic depreciation applies to all assets equally, and it does not consider the useful life. This is strange; is this a drawback of economic depreciation?

Answer: Just the opposite. Most accounting rules for depreciation are *ad hoc*. A building may be depreciated over 20 years, when its useful life is 50 years; machinery may be depreciated over 10 years when it will be replaced in 2 years; goodwill may have a 20 year schedule when it has no value; land may increase in value,

but depreciation doesn't show this; research and development has great value and immediate depreciation. The accounting rules are needed for *transparency* and *comparability*.

- *transparency:* users of financial statements understand the source of the numbers
- *comparability:* users can compare different firms

Economic depreciation measures the actual change in the value of the project's assets. The value of the assets depends on their future cash flows. The change in future cash flows applies to any type of asset, so all assets are depreciable.

Question: Suppose an asset does have a useful life of five years. Isn't GAAP depreciation with a five year useful life more sensible than economic depreciation?

Answer: If the asset has a useful life of five years, the cash flows extend for five years, and economic depreciation also occurs over five years.

Question: Must we know accounting depreciation for this course?

Answer: The on-line course emphasizes economic depreciation, not accounting depreciation. Know simple pro-rata depreciation, not the more complex methods.

Question: What about tax depreciation?

Answer: Tax depreciation affects the cash flows of the project, so it affects present values, economic income, and economic depreciation. The final exam for this course covers tax depreciation. The problems give pro-rata depreciation, a purchase price, a salvage value, and a tax rate, and ask about the effects on economic income. Other problems give two or more depreciation schedules as well as no depreciation and immediate write-offs and ask which method gives the highest economic income. Tax depreciation affects cash flows which affect economic depreciation, but tax depreciation is not the same as economic depreciation. This exercise does not include taxes; other practice problems on the discussion forum discuss tax depreciation.

Question: Which depreciation method gives the highest NPV?

Answer: All depreciation methods give the same total dollar amount of depreciation. Faster tax depreciation gives the quicker cash flows and the higher net present values. Immediate write-off gives the highest NPV; no depreciation gives the lowest NPV.

** Exercise 11.2: Economic Depreciation

(Adapted from question 29 of the Fall 97 Course 2 examination)

A project with an initial investment of $512 and the cash flows below has a net present value of zero and no residual value after the second year. All cash flows occur at the end of the year.

End of Year	Cash flows
1	400
2	300

A. What is the capitalization rate for the project?
B. What is the economic depreciation in years one and two?
C. What is the economic income in years one and two?

Part A: We solve for the capitalization rate as a quadratic equation:

$$-\$512 + \$400 / R + \$300 / R^2 = 0 \Rightarrow R = 1.25, \text{ so the capitalization rate is } 25\%.$$

Note: Final exam problems may require solving a quadratic equation; the arithmetic will be simple.

Question: This problem is simplified, since the net present value and the salvage value are zero. How would the problem change if they are not zero?

Answer: The problem is identical if we change

- Initial investment of Y and NPV of 0 to initial investment of Y – Z and NPV of Z.
- Salvage value of 0 and last year's cash flow of Q to salvage value of S and last year's cash flow of Q–S.

Part B: The present values of the project are

End of Year	*PV(Future Cash Flows)*
0	$\$400 / 1.25 + \$300 / 1.25^2 = \$512$
1	$\$300 / 1.25 = \$240$
2	0

- The economic depreciation in year 1 is $512 – $240 = $272.
- The economic depreciation in year 2 is $240 – 0 = $240.

Part C: The economic income is the cash flow minus the economic depreciation:

- The economic income in year 1 is $400 – $272.00 = $128.00
- The economic income in year 2 is $300 – $240 = $60.00

We verify these figures: $512 × 25% = $128 and $240 × 25% = $60

** Exercise 11.3: Return Measures

A project with an initial investment of $1,000 yields the following cash flows:

Year	1	2	3	4
Cash Flow	$100	$400	$300	$200

The cost of capital is 12% per annum. Straight line depreciation over four years is used for book accounting.

A. What is the internal rate of return?
B. What is the economic rate of return in year 3?
C. What is the book rate of return in year 3?

Part A: The total future cash flow is $1,000 and the investment is $1,000, so the internal rate of return is 0%.

Part B: The economic rate of return in every year (after the initial investment) is 12%.

Part C: The book depreciation in each year is $250. The book income in year 3 is $300 – $250 = $50. The value of the project at the beginning of year 3, after one half has been depreciated, is $500. The book rate of return in year 3 is $50 / $500 = 10%.