Corpfin module 11, Economic depreciation and economic income: practice problems
Brealey and Myers Chapter 12 "Agency problems, compensation, performance measurement"
(The attached PDF file has better formatting.)
Brealey and Myers contrast book depreciation and income with economic depreciation and income. Know how to calculate

- Book and economic depreciation and income.
- Net return on investment and economic value added from book and economic accounting entries.

Brealey and Myers note that the cash inflows from a project are often low in early years and high in later years. Know the effects on economic value added computed from book vs economic accounting entries.

The exercises have dialogues between Jacob and Rachel explaining the intuition for economic depreciation and economic income. The computations are shown in detail, though they are not difficult. Know how to create the tables showing book income and economic income.

Most final exam problems use simple scenarios: cash flows for 3 or 4 years, straight line book depreciation, and round numerical figures.
** Exercise 11.1: Economic value added and accounting bases
A project has an initial investment of $\$ 10$ million. GAAP uses straight line depreciation over five years with no salvage value.

- For simplicity, assume the corporate tax rate is zero.
- The opportunity cost of capital for the project is $10 \%$ per annum.
- The project returns $\$ 1$ million $\times t$ in years $t=1$ to 5 .
- After year 5 , the project has no salvage value.
A. What is the book capital each year, using GAAP entries?
B. Using GAAP entries, what is EVA in year 1?
C. Using GAAP entries, what is EVA in year 5?
D. Using economic accounting entries, what is EVA in year 1?
E. Using economic accounting entries, what is EVA in year 5?
F. If the expected cash inflow in year $\mathrm{t}=5$ is $\$ 6$ million instead of $\$ 5$ million, what is the change in book EVA, NPV, and economic EVA?
G. If the expected cash inflow in year $t=5$ is $\$ 5$ million and the actual cash inflow is $\$ 6$ million, what is the change in book EVA, NPV, and economic EVA?

Solution 11.1: The table below shows the computations (in thousands of dollars) using book entries.

|  | 1 | 2 | 3 | 4 | 5 | $\$$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow | $1,000.00$ | $2,000.00$ | $3,000.00$ | $4,000.00$ | $5,000.00$ | $o$ |
| Depreciation | $2,000.00$ | $2,000.00$ | $2,000.00$ | $2,000.00$ | $2,000.00$ | 2 |
| Book Value | $10,000.0$ | $8,000.00$ | $6,000.00$ | $4,000.00$ | $2,000.00$ | 0 |
| Book Income | $-1,000.0$ | 0.00 | $1,000.00$ | $2,000.00$ | $3,000.00$ | ( |
| Book ROI | -0.1000 | 0.0000 | 0.1667 | 0.5000 | 1.5000 | $?$ |
| EVA | $-2,000.0$ | -800.00 | 400.00 | $1,600.00$ | $2,800.00$ |  |
| Investment | $10,000.0$ | Years | 5.00 | Coc | 0.10 |  |

Part A: The book capital is $\$ 10$ million in year $1, \$ 8$ million in year $2, \ldots$, and $\$ 2$ million in year 5 . This is the line labeled "book value" in the table above.

Part B: The cash flow in year 1 is $\$ 1$ million, depreciation is $\$ 2$ million, and capital is $\$ 10$ million.
EVA using GAAP entries is $\$ 1$ million $-\$ 2$ million of depreciation $-\$ 10$ million $\times 10 \%=\$-2.00$ million.
Part C: The cash flow in year 5 is $\$ 5$ million, depreciation is $\$ 2$ million, and capital is $\$ 2$ million.
EVA using GAAP entries is $\$ 5$ million - $\$ 2$ million of depreciation $-\$ 2$ million $\times 10 \%=\$ 2.80$ million.

The table below shows the computations (in thousands of dollars) using book entries.

|  | 1 | 2 | 3 | 4 | 56 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow | 1,000.00 | 2,000.00 | 3,000.00 | 4,000.00 | 5,000.00 |
| Present value | 10,652.5 | 10,717.8 | 9,789.63 | 7,768.60 | 4,545.45 |
| Economic Depreciation | (65.26) | 928.22 | 2,021.04 | 3,223.14 | 4,545.45 |
| Economic Income | 1,065.26 | 1,071.78 | 978.96 | 776.86 | 454.55 d |
| Economic ROI | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 ? |
| EVA | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 d |
| Investment | 10,000.0 | Years | 5.00 | Coc | 0.10 |

Part D: Work out economic depreciation and economic income in year 1.

- The present value of the project at the beginning of year $1\left(\mathrm{PV}_{1}\right)$ is
$\$ 1$ million $/ 1.10^{1}+\$ 2$ million $/ 1.10^{2}+\$ 3$ million $/ 1.10^{3}+\$ 4$ million $/ 1.10^{4}+\$ 5$ million $/ 1.10^{5}=\$ 10.6526$ million
- The present value of the project at the end of year $1\left(\mathrm{PV}_{2}\right)$ is
$\$ 2$ million $/ 1.10^{1}+\$ 3$ million $/ 1.10^{2}+\$ 4$ million $/ 1.10^{3}+\$ 5$ million $/ 1.10^{4}=\$ 10.7178$ million
The economic capital is the worth of the project at the beginning of the year.
- Economic depreciation is year 1 is $\mathrm{PV}_{1}-\mathrm{PV}_{2}=10.6526$ million -10.7178 million $=\$-65.24$ thousand
- Economic income is $\$ 1$ million $-P V_{1}+P V_{2}=\$ 1,065.24$ thousand

Economic value added is economic income minus the net investment in the project (invested capital) times the opportunity cost of capital $=\$ 1,065.245-\$ 10,652.45 \times 10 \%=\$ 0$.

Question: The initial investment is $\$ 10,000$, not $\$ 10,652.45$.
Answer: The NPV of the projects is $\$ 10,652.45-\$ 10,000=\$ 652.45$. Investors receive this profit at time $t=0$ : if they sell the project, they receive this profit. This profit is re-invested in the project; the capital invested in the project the present value of future cash flows.

Part E: Work out economic depreciation and economic income in year 5.

- The present value of the project at the beginning of year 5 is $\$ 5$ million / $1.10=\$ 4,545.45$ thousand.
- The present value of the project at the end of year 5 is zero, since there are no future cash flows.
- Economic depreciation is year 5 is $\$ 4.545$ million.
- Economic income is $\$ 5$ million $-\$ 4.545$ million $=\$ 454.55$ thousand.

EVA is $\$ 5$ million $-\$ 4.545$ million $-\$ 4.545$ million $\times 10 \%=\$ 0.00$.
Question: If expected EVA is zero each year, then economic income $=$ present value of future cash flows $\times$ the opportunity cost of capital; is that correct?

Answer: Yes; you can always use this check. This is true for expected economic income; if cash flows are higher or lower than expected, EVA differs.

Part F: EVA based on book accounting entries increases by $\$ 1$ million (to $\$ 3.8$ million).

NPV increases by $\$ 1$ million $/ 1.10^{5}=\$ 620,921$.
EVA based on economic accounting entries remains zero.
Part G: EVA based on book accounting entries increases by $\$ 1$ million (to $\$ 3.8$ million).
NPV is based on expected values; it does not change.
EVA based on economic accounting entries becomes $\$ 1$ million.
[This exercise uses algebraic notation; most final exam problems give numerical values. Focus on the last part of this exercise: the effect of the discount rate on NPV, depreciation, economic income, and EVA.]
** Exercise 11.2: Economic value added
Use the following notation:

- $r=$ opportunity cost of capital
- $r_{f}=$ risk-free rate
- $c=$ capital invested in the project at time $\mathrm{t}=0$
- $c f_{0}=$ cash flow from time $\mathrm{t}=0$ to time $\mathrm{t}=1$
- $c f_{n}=$ cash flow from time $\mathrm{t}=\mathrm{n}$ to time $\mathrm{t}=\mathrm{n}+1$
- $p v_{0}=$ present value of the project at time $t=0$
- $p v_{1}=$ present value of the project at time $t=1$
- $p v_{n}=$ present value of the project at time $\mathrm{t}=\mathrm{n}$

The firm uses economic accounting to determine NPV, income, depreciation, and EVA. Assume that actual results equal expected results in each period.
A. What is the NPV of this project?
B. What is economic depreciation from time $t=0$ to time $t=1$ ?
C. What is economic income from time $t=0$ to time $t=1$ ?
D. What is economic value added from time $t=0$ to time $t=1$ ?
E. If the opportunity cost of capital for this project increases, what happens to NPV, economic depreciation, economic income, and economic value added?

Part A: To work out the NPV from the cash flows, one must know the dates of the cash flows.

- If the average cash flow occurs at mid-year, the NPV is $-c+\sum \mathrm{cf}_{\mathrm{j}} /(1+\mathrm{r})^{(0+1 / 2)}$, for $\mathrm{j}=0$ to $\infty$
- If the cash flows occur at the end of each year, the NPV is $-c+\sum \mathrm{cf}_{\mathrm{j}} /(1+\mathrm{r})^{j+1)}$, for $\mathrm{j}=0$ to $\infty$

In either case, the NPV is $-c+p v_{0}$.
Part B: Economic depreciation from time $\mathrm{t}=0$ to time $\mathrm{t}=1$ is $p v_{0}-p v_{1}=$ present value of the project at time $\mathrm{t}=0$ - present value of the project at time $t=1$.

Question: Depreciation is the decline in the value of buildings and equipment. What does this have to do with the change in present values?

Answer: Depreciation is the change in the value of assets from time $t=n$ to time $t=n+1$. The value of an asset is the present value of the future cash flows from that asset, so depreciation equals $p v_{n}-p v_{n+1}$.

Part C: Economic income from time $\mathrm{t}=0$ to time $\mathrm{t}=1$ is $\mathrm{cf}_{0}-\left(p v_{0}-p v_{1}\right)$.
Question: Income has many pieces besides cash flows and depreciation. If a firm buys inventory for $\$ 1,000$, its cash flow is $-\$ 1,000$ but its assets increase $\$ 1,000$, and its income is zero. Where is the change in net working capital, fixed assets, and liabilities in the equation for economic income?

Answer: An increase in net working capital or fixed assets increases the present value of the project, and an increase in liabilities decreases the present value of the project. Economic depreciation includes all the items that comprise accounting income except for cash flows.

Part D: Economic value added from time $\mathrm{t}=0$ to time $\mathrm{t}=1$ is economic income minus $\mathrm{r} \times$ invested capital, which is the present value of the project at time $t=0: c f_{0}-\left(p v_{0}-p v_{1}\right)-p v_{0} \times r$.

Question: The invested capital is $c$, not $p v_{0}$.
Answer: The invested capital for book accounting is the capital contributed by owners. The invested capital for economic accounting is the market value of the project.

Illustration: A firm has a factory that was built for $\$ 10$ million. It uses the factory for a project. The firm would sell the factory for $\$ 12$ million instead of using it for the project. The invested capital is $\$ 12$ million, not $\$ 10$ million. Conceive of this as though the firm sells the factory for $\$ 12$ million right before starting the project and buys it back for $\$ 12$ million. The factory is an investment of $\$ 12$ million, not $\$ 10$ million.

Part E: The NPV decreases.

- The exercise assumes the initial investment and future cash flows do not change.
- The present value of future cash flows is lower, since the discount rate is higher.

Economic depreciation decreases. To see this, consider a one dollar cash flow one year in the future.
Economic depreciation is (the present value of $\$ 1$ ) - $\$ 1$, which decreases as the discount rate increases.

- For a cash flow of N dollars, multiply both figures by N .
- For a cash flow k years in the future, multiply both figures by $(1+\mathrm{r})^{-(k-1)}$.

Economic income is the cash flow minus economic depreciation, so it increases.
EVA doesn't change if the higher capitalization rate is a higher expected capitalization rate.

## ** Exercise 11.3: Economic income and depreciation

A project has the following cash flows (time $j$ means "end of year $j$ "):

| Time | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Net Cash Flow | -100.00 | 0.00 | 78.55 | 78.55 |

The project's opportunity cost of capital is $20 \%$.
A. What is the present value of the project at the beginning of years 1,2 , and 3 ?
B. What is economic depreciation in years 1,2 , and 3 ?
C. What is economic income in years 1,2 , and 3 ?

Part A: Discount the cash flows at the $20 \%$ opportunity cost of capital.

- The present value at the beginning of year 3 is $\$ 78.55 / 1.20=\$ 65.46$.
- The present value at the beginning of year 2 is $(\$ 78.55+\$ 65.46) / 1.20=\$ 120.01$.
- The present value at the beginning of year 1 is $(\$ 120.01+\$ 0) / 1.20=\$ 100.01$.

Part B: Economic depreciation is the first differences of the present values.

- Year 1: $\$ 100.01-\$ 120.01=\$-20.00$.
- Year 2: $\$ 120.01-\$ 65.46=\$ 54.55$.
- Year 3: $\$ 65.46-0=\$ 65.46$.

Part C: Economic income is the cash flow minus economic depreciation.

- Year 1: $\$ 0.00-\$ 20.00=\$ 20.00$.
- Year 2: $\$ 78.55-\$ 54.55=\$ 24.00$.
- Year 3: $\$ 78.55-\$ 64.46=\$ 13.09$.

| Time | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Net Cash Flow | -100.00 | 0.00 | 78.55 | 78.55 |
| Present value (boty) | 0.00 | 100.01 | 120.01 | 65.46 |
| Econ Depreciation | -100.00 | -20.00 | 54.55 | 65.46 |
| Economic income | 0.00 | 20.00 | 24.00 | 13.09 |
| Capitalization rate | 0.20 |  |  |  |

** Exercise 11.4: Economic income and depreciation
A project has the following cash flows: an initial investment of $\$ 12$ million in plant and equipment and cash flows of $\$ 5.20$ million, $\$ 4.80$ million, and $\$ 4.40$ million in the three following years.

| Year | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Net Cash Flow | -12.00 | 5.20 | 4.80 | 4.40 |

The opportunity cost of capital is $10 \%$ per annum.
The firm uses straight-line depreciation. It writes off $\$ 4$ million each year in years 1,2 , and 3.
A. What is book depreciation each year?
B. What is book income each year?
C. What is the book capital invested in the project each year?
D. What is the book return on capital each year?
E. What is the present value of the project at each valuation date?
F. What is economic depreciation each year?
G. What is economic income each year?
H. What is EVA (economic value added) each year based on economic accounting?

Part A: The initial investment of $\$ 12$ million is amortized over three years, so book depreciation is $\$ 4$ million each year.

Part B: Book income is the cash flow minus book depreciation:

- Year 1: $\$ 5.20$ million $-\$ 4$ million $=\$ 1.2$ million
- Year 2: $\$ 4.80$ million $-\$ 4$ million $=\$ 0.8$ million
- Year 3: $\$ 4.40$ million $-\$ 4$ million $=\$ 0.4$ million

Question: What is book income in year $0 ?$
Answer: The cash flow is $-\$ 12$ million and book depreciation is $-\$ 12$ million, so book income is zero. The depreciation over the entire project is zero, since assets before inception are zero and assets after the project ends are zero. Department at inception is the negative of depreciation in subsequent years.

Part C: The book capital invested in the project is the initial investment minus the depreciation before the year.

- Year 1: $\$ 12$ million - $\$ 0=\$ 12$ million
- Year 2: $\$ 12$ million - $\$ 4$ million $=\$ 8$ million
- Year 3: $\$ 12$ million $-\$ 8$ million $=\$ 4$ million

Part D: The book return on capital is book income divided by book capital:

- Year 1: $\$ 1.2$ million $/ \$ 12$ million $=10 \%$
- Year 2: $\$ 0.8$ million / $\$ 8$ million $=10 \%$
- Year 3: $\$ 0.4$ million $/ \$ 4$ million $=10 \%$

Part E: The present value of the project is the present value of future cash flows.

- Time $t=0: \$ 5.20$ million $/ 1.10^{1}+\$ 4.8$ million $/ 1.10^{2}+\$ 4.4$ million $/ 1.10^{3}=\$ 12,000,000$
- Time $t=1: \$ 4.8$ million $/ 1.10^{1}+\$ 4.4$ million $/ 1.10^{2}=\$ 8,000,000$
- Time $t=2: \$ 4.4$ million $/ 1.10^{1}=\$ 4,000,000$

Economic depreciation is the change in the present value of future cash flows over the course of the year.

- Year 1: $\$ 12$ million $-\$ 8$ million $=\$ 4$ million
- Year 2: $\$ 8$ million $-\$ 4$ million $=\$ 4$ million
- Year 3: $\$ 0$ million - $\$ 0$ million $=\$ 4$ million

Part F: Economic income is the cash flow minus economic depreciation:

- Year 1: $\$ 5.20$ million $-\$ 4$ million $=\$ 1.2$ million
- Year 2: $\$ 4.80$ million $-\$ 4$ million $=\$ 0.8$ million
- Year 3: $\$ 4.40$ million - $\$ 4$ million $=\$ 0.4$ million

Part G: EVA (using economic accounting) is economic income minus the cost of capital times invested capital.

- Year 1: $\$ 1.2$ million $-\$ 12$ million $\times 10 \%=0$
- Year 2: $\$ 0.8$ million $-\$ 8$ million $\times 10 \%=0$
- Year 3: $\$ 0.4$ million $-\$ 4$ million $\times 10 \%=0$

Question: Is it usual to have zero EVA each year?
Answer: The expected EVA each year using economic accounting is always zero. The entire economic value of the project is recognized at inception: the net present value.
** Exercise 11.5: Economic income and depreciation
An auto leasing project has a four year life. It uses straight line book depreciation over four years to a 20\% salvage value, when the used car will be sold in the second hand market.

Consumers prefer newer cars. As a car ages, it produces less cash flow, since it is leased less often.

- Each car costs $\$ 50,000$ at time $t=0$.
- Cash inflows are $\$ 40,000$ at time $t=1, \$ 30,000$ at time $t=2, \$ 20,000$ at time $t=3$, and $\$ 10,000$ at time $t=4$.
- Auto leasing is a high risk venture, with a CAPM beta of 1.25.
- The risk-free rate is $4 \%$ per annum and the market risk premium is $8 \%$.
A. What is the opportunity cost of capital for this project?
B. What is book depreciation each year and the salvage value at the end?
C. What is the NPV of the project? (Do with and without taxes)
D. Suppose (GAAP or) tax law allows depreciation over two years instead of four years (to a zero salvage value). What is new NPV?
E. What is book income each year? 30, 20, 10, 0 ; also show after-tax
F. What is the present value of the project each year? pv's are $40 / 1.14^{\wedge} 1+30 / 1.14^{\wedge} 2 \ldots$
G. What is economic depreciation each year? first differences
H. What is economic income each year? add cash flows
I. What is the economic value added each year?

Part A: The opportunity cost of capital is $4 \%+1.25 \times 8 \%=14 \%$.
Part B: The salvage value is $20 \% \times \$ 50,000=\$ 10,000$. The remaining $\$ 40,000$ is depreciated over four years at $\$ 10,000$ a year.

Part C: The NPV of the project is
$-\$ 50,000+\$ 40,000 / 1.14^{1}+\$ 30,000 / 1.14^{2}+\$ 20,000 / 1.14^{3}+\$ 10,000 / 1.14^{4}+\$ 10,000 / 1.14^{4}=\$ 33,512.78$
Question: Why are there two terms for $\$ 10,000 / 1.14^{4}$ in this expression?
Answer: The first term is the cash inflow at time $t=4$. The second term is the sale of the used auto. They have the same cash flow by coincidence

Question: What about the depreciation each year?
Answer: Tax depreciation affects the tax cash flows, so it is part of the NPV formula. Book depreciation does not affect the cash flows, so it is ignored in the NPV formula. If the tax rate is zero, book depreciation does not affect net present value.

Part C: With taxes, depreciation causes a tax refund. We ignore book depreciation, which is not a cash flow, and use tax depreciation, which causes tax cash flows. The after-tax cash flows are

- time $t=0$ : $-\$ 50,000$
- time t=1: $\$ 40,000-35 \% \times(\$ 40,000-\$ 10,000)$
- time $t=2: \$ 30,000-35 \% \times(\$ 30,000-\$ 10,000)$
- time $t=3: \$ 20,000-35 \% \times(\$ 20,000-\$ 10,000)$
- time $t=4: \$ 10,000-35 \% \times(\$ 10,000-\$ 10,000)+\$ 10,000$

Question: What about the tax refund on the $\$ 50,000$ purchase of the car?

Answer: Taxes are paid on profits, not on cash flows.
At time $t=0$, the firm pays $\$ 50,000$ for the car. Tax depreciation starts the next year, so the tax value of the car is $\$ 50,000$. The firm has no profit or loss, so the tax liability or refund is zero.

Question: What about the tax liability on the $\$ 10,000$ resale value of the car?
Answer: At time $t=4$, the firm sells the car for its book value of $\$ 10,000$. The cash inflow is $\$ 10,000$, but the gain or loss is zero, so the firm has no tax liability.

Question: The textbook speaks of after-tax cash flows and depreciation tax shields. How do we show these?
Answer: Rewrite the after-tax cash flows as

- time $\mathrm{t}=0$ : $-\$ 50,000$
- time $\mathrm{t}=1: \$ 40,000-35 \% \times(\$ 40,000-\$ 10,000)=(1-35 \%) \times \$ 40,000+35 \% \times \$ 10,000$
- time $\mathrm{t}=2$ : $\$ 30,000-35 \% \times(\$ 30,000-\$ 10,000)=(1-35 \%) \times \$ 30,000+35 \% \times \$ 10,000$
- time $\mathrm{t}=3: \$ 20,000-35 \% \times(\$ 20,000-\$ 10,000)=(1-35 \%) \times \$ 20,000+35 \% \times \$ 10,000$
- time $\mathrm{t}=4: \$ 10,000-35 \% \times(\$ 10,000-\$ 10,000)+\$ 10,000=(1-35 \%) \times \$ 10,000+35 \% \times \$ 10,000+$ \$10,000

Question: At time $\mathrm{t}=1$, the after-tax cash flow is $\$ 40,000-35 \% \times(\$ 40,000-\$ 10,000)=(1-35 \%) \times \$ 40,000$ $+35 \% \times \$ 10,000$. What is the rationale for this expression?

Answer: The first expression is the pre-tax cash flow $(\$ 40,000)$ minus the tax rate $(35 \%) \times$ taxable income (\$40,000 - \$10,000).
The second expression after-tax income (ignoring depreciation) plus the depreciation tax shield.
Question: The first and last cash flows also have terms with no tax rate.
Answer: The purchase of the auto ( $-\$ 50,000$ ) and the sale of the auto $(+\$ 10,000)$ are not taxed, but they are cash flows. Buying the auto exchanges cash for an asset (autos), each of which is worth $\$ 50,000$; it does not affect taxable income. Selling the auto for its salvage value is the same, except now the car is worth $\$ 10,000$.

Part D: GAAP (book) depreciation does not affect net present value. Accelerated depreciation does not affect NPV.

Part D: With accelerated tax depreciation, tax depreciation is higher at times $\mathrm{t}=1$ and 2 and lower at times $\mathrm{t}=3$ and 4 , so the tax liability is lower at times $\mathrm{t}=1$ and 2 and higher at times $\mathrm{t}=3$ and 4 . The pre-tax cash flows are the same as before, but the after-tax cash flows are higher at times $t=1$ and 2 and lower at times $t=3$ and 4 .

Total depreciation is the same, so the firm has the same total cash inflow, but the after-tax cash inflows are higher at times $\mathrm{t}=1$ and 2 and lower at times $\mathrm{t}=3$ and 4 . Book EVA is higher in years 1 and 2 and lower in years 3 and 4.

## Question: How does economic EVA change?

If the faster depreciation is known at inception of the project, the NPV of the project is higher, and the expected EVA each year remains zero.

Part E: Book income is the cash flow minus book depreciation:

- Year 1: $\$ 40,000-\$ 10,000=\$ 30,000$
- Year 2: $\$ 30,000-\$ 10,000=\$ 20,000$
- Year 3: $\$ 20,000-\$ 10,000=\$ 10,000$
- Year 4: $\$ 10,000-\$ 10,000=\$ 0$

Part F: After-tax book income equals taxable income times (1 - the tax rate).

- Year 1: $(1-35 \%) \times(\$ 40,000-\$ 10,000)=\$ 19,500$
- Year 2: $(1-35 \%) \times(\$ 30,000-\$ 10,000)=\$ 13,000$
- Year 3: $(1-35 \%) \times(\$ 20,000-\$ 10,000)=\$ 6,500$
- Year 4: $(1-35 \%) \times(\$ 10,000-\$ 10,000)=\$ 0$
\{version 2: these are \&pt \&cfs. \&tr is $35 \%$. tax \&depr is straight line oer four years, like boik \&depr.)
\{version 3: auto leasing profits depend on economic cycles. assume projected pv's with no econmic cycle, but actual \&cfs are 10, 20, 30, 40\}
** Exercise 11.6: ROI, EVA, and opportunity cost of capital
- $\mathrm{ROI}=$ (gross) return on investment (not considering the opportunity cost of capital)
- COC = opportunity cost of capital (market capitalization rate for the project)
- EVA = economic value added
- INV = net investment in the project
- net ROI = net return on investment (after considering the opportunity cost of capital)

Assume all items are computed from economic accounting entries.
A. If $\mathrm{ROI}=\mathrm{COC}$, what is the EVA?
B. If $\mathrm{ROI}=2 \times \mathrm{COC}$, what is the EVA?
C. If $\mathrm{ROI}=0$, what is the EVA?
D. If $\mathrm{ROI}=2 \times \mathrm{COC}$, what is the net ROI ?
E. If $\mathrm{ROI}=0$, what is the net ROI ?

Part A: If ROI = COC, the EVA is zero.

- Economic income is the $\mathrm{ROI} \times$ the net investment.
- EVA is income minus the COC $\times$ the net investment.

Part B: If $\mathrm{ROI}=2 \times \mathrm{COC}$, the $\mathrm{EVA}=\mathrm{COC} \times \mathrm{INV}$.
Part C: If $\mathrm{ROI}=0$, the $\mathrm{EVA}=-C O C \times I N V$,
Part D: Net ROI $=$ gross $\mathrm{ROI}-\mathrm{COC}$. If $\mathrm{ROI}=2 \times \mathrm{COC}$, net $\mathrm{ROI}=\mathrm{COC}$.
Part E: If $\mathrm{ROI}=0$, net $\mathrm{ROI}=-\mathrm{COC}$.
** Exercise 11.7: Economic value added and accounting entries
A project uses straight line depreciation for its GAAP financial statements.

- The opportunity cost of capital is $12 \%$ per annum, and the project's NPV is $\$ 100,000$.
- The cash inflows from the project increase each year for five years.
- The project has no salvage value after five years.
- Book EVA = EVA computed from book accounting entries.
- Economic EVA = EVA computed from economic accounting entries.
A. What is the pattern of expected book EVA by year?
B. What is the pattern of expected economic EVA by year?

Part A: Expected book EVA increases each year.

- EVA is net income minus the cost of capital times the net investment.
- Net income is the cash inflow minus depreciation.

For two reasons, expected book EVA increases each year.

1. In this exercise, book depreciation is constant. We don't know the annual depreciation, since we don't know the cash inflows each year, but straight line depreciation means equal amounts each year. The cash inflow increases each year, so net income increases each year.
2. The expected opportunity cost of capital is the same each year. The net investment is the undepreciated value of the assets, which declines each year.

Part B: Expected economic EVA is zero each year by definition.
Economic depreciation is the change in the present value of the project: $P V_{t-1}-P V_{t}$. Economic income is the cash inflows minus economic depreciation $=C F+P V_{t}-P V_{t-1}$. This is the return on capital, or $R O I \times \mathrm{PV}_{\mathrm{t}-1}$.

The expected return on capital is the opportunity cost of capital. EVA subtracts $\mathrm{OCOC} \times \mathrm{PV}_{\mathrm{t}-1}$ from economic income. This gives a zero expected EVA each year.

Question: If EVA is zero each year, what use is it?
Answer: Expected EVA is zero each year, not actual EVA. A firm can do better (or worse) than expected many ways. If the cash inflows are higher than expected one year, the EVA is positive. If the market for the product improves, and the present value of the future cash flows increases, economic depreciation is negative and EVA is higher than zero.

Question: EVA is economic value added. How can one compute EVA from book accounting entries? Isn't this mixing apples and oranges?

Answer: EVA subtracts the dollar cost of capital from net income. Both net income and the dollar cost of capital can be determined from book accounting entries or economic accounting entries. Most managers know their book accounting figures; computing economic value added from these book accounting entries is better than simple net income. Brealey and Myers recommend economic accounting entries, since book accounting may distort some figures, but most firms do not use economic accounting.

Net income is cash flows minus depreciation, at least for the simplified firms in the textbook examples. Many firms that use EVA subtract book depreciation from the cash flows. This distorts the EVA measure if the worth
of the assets does not vary with book depreciation. But the distortion is not material if depreciation schedules match the changes in the value of the assets,

Brealey and Myers use economic depreciation. This type of depreciation can be viewed two ways.

- Economic depreciation is the amount of depreciation that causes expected EVA to be zero in each year.
- Economic depreciation is the change in the present value of the project.

Brealey and Myers defend these views of economic depreciation. But some managers do not accept their reasoning, and think their logic is circular. For the final exam problems, understand EVA from both a book accounting and an economic accounting perspective.

Question: Does the dollar cost of capital also differ between book and economic accounting?
Answer: For book accounting, capital is the undepreciated value of the assets. Many firms use accelerated depreciation schedules to minimize taxes. These depreciation schedules distort the EVA. If a firm's assets are fully depreciated over the first ten years, but their useful lives are 20 years, the firm subtracts nothing from net income to form EVA in the second ten years.

For economic accounting, capital is the present value of the project. This makes more sense. If the firm can sell the project for its present value, then the capital tied up in the project is this present value. Capital remains positive as long as the project has a positive present value.
** Exercise 11.8: Growth
Walmart is expanding in many countries. New Walmart stores have high (but declining) investment expenses the first few years. They have low cash inflows the first few years, but cash inflows increase as consumer learn the value of shopping at Walmart.

In 20X1, Walmart plans to grow faster, to match higher growth rates of emerging economies. The expected cash flow pattern of each new store remains the same, but Walmart plans to open more new stores each year.
A. What is the expected effect of faster growth on EVA computed from book accounting entries?
B. What is the expected effect of faster growth on EVA computed from economic accounting entries?

Part A: For any store, expected book EVA is negative in early years (when net investment is high and cash inflows are low) and positive in later years (when net investment is low and cash inflows are high). In a steady state, the new stores offset the older stores, and overall expected EVA is zero. If the firm's growth rate accelerates, there are more new stores as a percentage of total stores, so the overall EVA declines. Book EVA will understate true profitability.

Part B: For each store, expected EVA computed from economic accounting entries is zero each year. The growth rate does not affect the expected EVA.
** Exercise 11.9: Economic value added
A project has an initial investment of $\$ 1,000$, a six year life with no salvage value, and expected cash flows at the end of these six years of $\$ 100, \$ 200, \$ 250, \$ 298, \$ 298$, and $\$ 297$. The initial investment is depreciated straight line over six years. The opportunity cost of capital is 10\%. Using book accounting entries:
A. What is depreciation each year?
B. What is the present value of the project at the beginning of each year ("book value")?
C. What is book income each year?
D. What is the book return on investment (ROI) each year?
E. What is the economic value added (EVA) each year?

Solution 11.9: For the table below. Final exam problems use fewer years, so the computations are easy.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow | 100.00 | 200.00 | 250.00 | 298.00 | 298.00 | 297.00 |
| Depreciation | 166.67 | 166.67 | 166.67 | 166.67 | 166.67 | 166.67 |
| Book Value | $1,000.00$ | 833.33 | 666.67 | 500.00 | 333.33 | 166.67 |
| Book Income | -66.67 | 33.33 | 83.33 | 131.33 | 131.33 | 130.33 |
| Book ROI | -0.0667 | 0.0400 | 0.1250 | 0.2627 | 0.3940 | 0.7820 |
| EVA | -166.67 | -50.00 | 16.67 | 81.33 | 98.00 | 113.67 |

Part A: Book depreciation is the initial investment divided by the number of years: $\$ 1000 / 6=\$ 166.67$.
Part $B$ : The book value of the project is the initial investment minus cumulative depreciation.

- At the start of year 1, nothing has yet been depreciated, so the book value of the project is $\$ 1,000$.
- At the start of year 2, cumulative depreciation is $\$ 166.67$, so the book value of the project is $\$ 1,000-$ \$166.67 = \$833.33.

Part C: Book income is the cash flow minus depreciation.

- Year 1: $\$ 100-\$ 166.67=-\$ 66.67$
- Year 2: $\$ 200-\$ 166.67=+\$ 33.33$

Part D: Book ROI is book income divided by book value.

- Year 1: $-\$ 66.67 / \$ 1,000=-0.0667$
- Year 2: $+\$ 33.33 / \$ 833.33=0.0400$

Part E: EVA is book income minus book value $\times$ the opportunity cost of capital.

- Year 1: $-\$ 66.67-\$ 1,000 \times 10 \%=-166.67$
- Year 2: $+\$ 33.33-\$ 833.33 \times 10 \%=-50.00$

Note: For book accounting, work out book values from the previous book value and depreciation.
** Exercise 11.10: Economic value added
A project has an initial investment of $\$ 1,000$, a six year life with no salvage value, and expected cash flows at the end of these six years of $\$ 100, \$ 200, \$ 250, \$ 298, \$ 298$, and $\$ 297$. The initial investment is depreciated straight line over six years. The opportunity cost of capital is $10 \%$. Using economic accounting entries:
A. What is the present value of the project at the beginning of each year ("book value")?
B. What is depreciation each year?
C. What is book income each year?
D. What is the book return on investment (ROI) each year?
E. What is the economic value added (EVA) each year?

Solution 11.10: For the table below. Final exam problems use fewer years, so the computations are easy.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow | 100.00 | 200.00 | 250.00 | 298.00 | 298.00 | 297.00 |
| Present value | $1,000.25$ | $1,000.27$ | 900.30 | 740.33 | 516.36 | 270.00 |
| Economic Depreciation | $(0.02)$ | 99.97 | 159.97 | 223.97 | 246.36 | 270.00 |
| Economic Income | 100.02 | 100.03 | 90.03 | 74.03 | 51.64 | 27.00 |
| Economic ROI | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 |
| EVA | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Investment | $1,000.00$ | Years | 6.00 | Coc | 0.10 |  |

Part A: The present value of the project is the present value of future cash flows.

- At the start of year 6, \$297 / $1.10=\$ 270.00$
- At the start of year $5, \$ 298 / 1.10+\$ 297 / 1.10^{2}=\$ 516.36$

Part B: Economic depreciation is the first differences of successive present values of the project

- Year 1: \$1,000.25-\$1,000.27 = - $\$ 0.02$
- Year 2: $\$ 1,000.27-\$ 900.30=+\$ 99.97$

Part C: Economic income is the cash flow minus depreciation.

- Year 1: $\$ 100--\$ 0.02=\$ 100.02$
- Year 2: $\$ 200-\$ 99.97=+\$ 100.03$

Part D: Economic ROI is economic income divided by present value at the beginning of the year.

- Year 1:\$100.02 / \$1,000.25 $=0.1000$
- Year 2: $\$ 100.03 / \$ 1,000.27=0.1000$

Part E: EVA is economic income minus present value $\times$ the opportunity cost of capital.

- Year 1: $\$ 100.02-\$ 1,000.25 \times 10 \%=\$ 0.00$
- Year 2: $\$ 100.03-\$ 1,000.27 \times 10 \%=\$ 0.00$

Note: For economic accounting, work out depreciation from first differences of present values of the project.
** Exercise 11.11: Economic value added
A project has an initial investment of $\$ 1,000$, a five year life with no salvage value, an opportunity cost of capital of $12 \%$, and expected cash flows at the end of these five years of $\$ 100, \$ 200, \$ 300, \$ 400$, and $\$ 500$. The initial investment is depreciated straight line over five years. Using book accounting entries:
A. What is depreciation each year?
B. What is the present value of the project at the beginning of each year ("book value")?
C. What is book income each year?
D. What is the book return on investment (ROI) each year?
E. What is the economic value added (EVA) each year?
F. In which years do book income, book ROI, and book EVA understate or overstate true earnings?

Solution 11.11: Form the table below.

|  | 1 | 2 | 3 | 4 | 5 | $\$ \mid$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow | 100.00 | 200.00 | 300.00 | 400.00 | 500.00 | ( |
| Depreciation | 200.00 | 200.00 | 200.00 | 200.00 | 200.00 | 2 |
| Book Value | $1,000.00$ | 800.00 | 600.00 | 400.00 | 200.00 | ? |
| Book Income | -100.00 | 0.00 | 100.00 | 200.00 | 300.00 | ( |
| Book ROI | -0.1000 | 0.0000 | 0.1667 | 0.5000 | 1.5000 | $?$ |
| EVA | -220.00 | -96.00 | 28.00 | 152.00 | 276.00 |  |
| Investment | $1,000.00$ | Years | 5.00 | Coc | 0.12 |  |

Part A: Book depreciation is the initial investment divided by the number of years: $\$ 1000 / 5=\$ 200$.
Part B: The book value of the project is the initial investment minus cumulative depreciation.

- At the start of year 1, nothing has yet been depreciated, so the book value of the project is $\$ 1,000$.
- At the start of year 2, cumulative depreciation = \$200; the book value $=\$ 1,000-\$ 200=\$ 800$.

Part C: Book income is the cash flow minus depreciation.

- Year 1: $\$ 100-\$ 200=-\$ 100$
- Year 2: $\$ 200-\$ 200=\$ 0$

Part D: Book ROI is book income divided by book value.

- Year 1: $-\$ 100 / \$ 1,000=-0.0100$
- Year 2: $\$ 0 / \$ 800=0.0000$
- Year 3: $\$ 100 / \$ 600=0.1667$

Part E: EVA is book income minus book value $\times$ the opportunity cost of capital.

- Year 1: $-\$ 100-\$ 1,000 \times 12 \%=\$-220.00$
- Year 2: $\$ 0-\$ 800 \times 12 \%=-96.00$

Part F: ROI should be 12\% each year, and EVA should be zero each year.

- In the first two years, book ROI is negative or zero and EVA is negative, so they understate true earnings.
- In the last three years, book ROI is more than $12 \%$ and EVA is positive, so they overerstate true earnings.

Question: What do you mean by true earnings? ROI and EVA measure performance; how can they overstate or understate anything?

Answer: Accounting systems allocate earnings to periods. GAAP has many arbitrary rules. In this exercise, straight line depreciation reduces earnings too much in early period and too little in later periods.

True earnings = the cash flow plus the change in market value. The market uses economic values, not book values. Book accounting does not reflect the true change in vaule.

Question: You say that book accounting has too much depreciation in early years and too little in later years. But the opposite is true. Suppose a new computer system costs $\$ 10$ million and will last ten years. After one year, the used computer system can be sold for $\$ 5$ million, not its book value of $\$ 9$ million. Its value may go down to $\$ 1$ million by the end of the third year and then decrease to zero slowly over the next seven years. Straight line depreciation is too low in early years and too high in later years.

Answer: The resale value of the equipment is not the relevant measure of the project's market value. In the first year, the firm learns how to use the new computer system. The cash inflow is small the first year, but the firm gains expertise, which adds to its intangible franchise value. The franchise value is not included in book value or book income, but it is part of economic value and economic income.

Note: For book accounting, work out book values from the previous book value and depreciation.
Note: Final exam problems may give the cash flow pattern and the market values of the equipment each year. Straight line depreciation overstates the market value of the equipment each year, so one might be tempted to say it overstates the ROI and the EVA each year. Economic ROI and EVA use the value of the project, not the resale value of the equipment.
** Exercise 11.12: Economic value added
A project has an initial investment of $\$ 1,000$, a five year life with no salvage value, an opportunity cost of capital of $12 \%$, and expected cash flows at the end of these five years of $\$ 100, \$ 200, \$ 300, \$ 400$, and $\$ 500$. Using economic accounting entries:
A. What is the present value of the project at the beginning of each year ("present value")?
B. What is depreciation each year?
C. What is book income each year?
D. What is the book return on investment (ROI) each year?
E. What is the economic value added (EVA) each year?

Solution 11.12: Form the table below.

|  | 1 | 2 | 3 | 4 | 5 | $\varepsilon$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash Flow | 100.00 | 200.00 | 300.00 | 400.00 | 500.00 | ¢ |
| Present value | 1,000.18 | 1,020.20 | 942.62 | 755.74 | 446.43 | q |
| Economic Depreciation | (20.02) | 77.58 | 186.89 | 309.31 | 446.43 | 0 |
| Economic Income | 120.02 | 122.42 | 113.12 | 90.69 | 53.57 |  |
| Economic ROI | 0.1200 | 0.1200 | 0.1200 | 0.1200 | 0.1200 |  |
| EVA | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | ¢ |
| Investment | 1,000.00 | Years | 5.00 | Coc | 0.12 |  |

Part A: The present value of the project is the present value of future cash flows.

- At the start of year $5, \$ 500 / 1.12=\$ 446.43$
- At the start of year $4, \$ 400 / 1.12+\$ 400 / 1.12^{2}=\$ 755.74$

Part B: Economic depreciation is the first differences of successive present values of the project

- Year 1: $\$ 1,000.18-\$ 1,020.20=\$-20.02$
- Year 2: $\$ 1,000.27-\$ 900.30=+\$ 99.97$

Question: Economic depreciation is negative the first year. Does that make sense?
Answer: This project's cash flows increase from year to year, since consumers need time to learn of the firm's products. If the cash flow in the year is small, the present value of the project increases over the year, so economic depreciation is negative.

Part C: Economic income is the cash flow minus depreciation.

- Year 1: $\$ 100--\$ 20.02=\$ 120.02$
- Year 2: $\$ 200-\$ 77.58=\$ 122.42$

Part D: Economic ROI is economic income divided by present value at the beginning of the year.

- Year 1:\$120.02 / \$1,000.18 = 0.1200
- Year 2: $\$ 122.42$ / \$1,020.20 = 0.1200

Part E: EVA is economic income minus present value $\times$ the opportunity cost of capital.

- Year 1: $\$ 120.02-\$ 1,000.18 \times 12 \%=\$ 0.00$
- Year 2: $\$ 122.42-\$ 1,020.20 \times 12 \%=\$ 0.00$

Note: For economic accounting, work out depreciation from first differences of present values of the project.
Question: This project has an NPV of $18 \phi$. Is the EVA zero each year because the NPV is zero?
Answer: The NPV of the project does not affect the EVA each subsequent year. The EVA gives the net income at inception of the project. The initial investment does not affect each year's EVA.

