Corporate Finance, Module 17, “Optimal Corporate Borrowing” (chapter 18)

Practice Problems

(The attached PDF file has better formatting.)

Question 17.1: Capital Restructuring

The market value of a firm financed solely with equity is $700 million. The firm converts some of its equity to debt yielding 6% per annum, changing the debt-to-equity ratio to 25%. (A 25% debt-to-equity ratio means that debt is 20% of total capital and equity is 80% of total capital.)

The corporate tax rate is zero, the personal tax rate on interest income is the same as on equity income, and there are no costs of bankruptcy or other market imperfections. (These assumptions mean that we can use the MM Proposition 1.)

A. How much equity is converted to debt? What are the revised values of E and D?
B. What is the value of the firm after the refinancing?
C. If the corporate tax rate is 35% and the same amount of equity is converted to debt, what is the present value of the tax shields created by the debt? Assume the debt is perpetual.
D. The tax shield affects the value of A, not of D. What is the new value of the firm?
E. What is the new value of equity? (E = A – D)
F. With the new value of E, the debt-to-equity ratio is no longer 25%. What is the new debt-to-equity ratio?
G. Suppose the firm desires a 25% debt-to-equity ratio. How much equity (in total) must be converted to debt? Solve this algebraically: Total capital = original capital of $700 million plus the tax shield, which is 35% of the debt: D = 20% × ($700 + 35% × D)
H. What is the final value of the firm?

Solution 17.1:

Part A: A 25% debt-to-equity ratio means that debt is 20% of total capital = 20% × $700 million = $140 million. The revised equity is 80% × $700 million = $560 million.

Part B: The value of the firm does not change by refinancing in perfect capital markets; this is the Modigliani and Miller Proposition 1.

Part C: $140 million of debt creates $140 million × 35% = $49 million of tax shields.

Question: The problem assumes that the debt is perpetual. But even long-term debt matures in 30 years; is it reasonable to assume that the debt is perpetual?

Answer: We do not mean that this bond indenture is perpetual, but that the firm intends to continue with a 25% debt-to-equity ratio indefinitely. This is reasonable: most firms which borrow roll over the debt when it matures.

Question: Is this a necessary assumption? What if the firm takes a ten year loan and does not intend to roll it over?

Answer: If the debt has a definite term and will not be rolled over, the present value of the future tax shields is lower. Finite term debt makes sense if it finances a limited term project. If the debt finances the firm as a whole, and if it is optimal for the firm to borrow now, it will probably be optimal for the firm to continue borrowing ten years from now. Brealey and Myers are not clear about how to value a firm that would be more valuable if it borrowed but whose present CEO does not want to borrow.

- Some analysts say that the firm will borrow as soon as the CEO retires or realizes the value of borrowing.
- In practice, some firms do not borrow even though we think they would gain much from a moderate amount of debt. We don't have a good explanation why some firms are so reluctant to borrow, but we explain the real world, not our view of the ideal world.

Part D: The revised value of the firm, including the tax shields, is A = $749 million.

Part E: The new value of equity is $749 million – $140 million = $609 million.

Part F: The new debt-to-equity ratio is $140 million / $609 million = 22.99% ≈ 23%.

Part G: Since the equity is larger, we can add more debt before reaching a 25% debt to equity ratio.

We solve for D (the debt) algebraically. The debt-to-equity ratio is 25%, so the debt to capital ratio is 25% / 125% or 20%. The total capital is the original capital of $700 million plus the value of the tax shield, which is 35% of the debt:

$$D = 20\% \times (\$700 + 35\% \times D)$$
$$93\%\, D = \$140$$
$$D = \$140 / 93\% = \$150.54$$

Part H: The value of the firm is 5 × $150.54 = $752.69.

Question 17.2: Tax Shields

A firm issues a $1,000 five-year annual coupon 8% bond at par value with a face value of $1,000. The corporate tax rate is 35%.

A. What are the cash flows (the coupon payments) of this bond?
B. What is the annual tax shield from this bond?
C. What is the present value of the interest tax shields generated by this bond?

Solution 17.2:

Part A: The interest payment is $1,000 × 8% = $80 each year.

Part B: At the 35% tax rate, the tax shield is $80 × 35% = $28 each year.

Part C: At an 8% discount rate, the present value of the tax shields is $28 × $(v + v^2 + v^4 + v^4 + v^5)$ = $111.80, where v = 1/1.08.

Question: Why is the tax shield worked out differently in this problem than in the previous problem?

Answer: This problem assumes five year debt that is *not* rolled over. The debt finances a five year project, which is not renewed.

Exercise 17.3: Tax Shields

Bonds Y and Z trade at par; their market value equals their par value, which are the same.

- Bond Y has 8% annual coupons and an 8% effective annual yield.
- Bond Z has 6% annual coupons and an 6% effective annual yield.

A. What are the annual tax shields for each bond?
B. If the bonds are perpetuities (i.e., they have perpetual durations), what are the present values of the annual tax shields for each bond? Which bond has the higher present value of the annual tax shields?
C. If the bonds have 1 year durations, what are the present values of the tax shield for each bond? Which bond has the higher present value of the tax shield?

Solution 17.3:

Question: Why do the bonds have different coupons? Are the bonds issued by different firms with different credit ratings?

Answer: Brealey and Myers are not clear about how to deal with firms with different credit ratings. Some bondholders have an 8% bond from firm A and a 15% bond from firm B. Firm B has a lower credit rating, so it has a higher coupon payment. But the bondholders do not expect to collect 7% more from Firm B. The higher default probability of Firm B means the probability is higher than the bondholders will not get paid. Most of the higher coupon rate covers the higher expected defaults; only a small part covers risk.

Question: What is the difference between expected defaults and risk?

Answer: Expected defaults are the cost of not getting paid. If bondholders were risk neutral, the coupon rate for Firm B would just cover the higher expected defaults, and the expected return to the bondholders would be the same for the two firms. The risk is the additional return required by bondholders for taking an uncertain investment.

Illustration: Two firms seek a one year loan.

- The bank is certain that Firm #1 will not default, and it charges an 8% risk-free rate.
- The bank believes that Firm #2 has a 10% chance of defaulting. The appropriate interest rate is $(1 + r) \times 90\% = 1.08 \Rightarrow r = 1.08 / 90\% - 1 = 20\%$.
- If the probability of default is correlated with market returns, this risk is systematic risk, and the bank requires a higher expected return. It may charge a rate of 22%: 8% is the risk-free rate, 12% is expected defaults, and 2% is a risk charge.

Question: Brealey and Myers apply the CAPM to both equity and debt returns. If the risk-free rate is r_f, the market risk premium is MRP, and the return on a firm's debt is r_d, the beta of the firm's debt is $\beta_d = (r_d - r_f) / MRP$. Suppose the risk-free rate here is 8% and the market risk premium is 7%. Is the beta of Firm #2's debt $(22\% - 8\%) / 7\% = 2$?

Answer: Firm #2 pays 22%, but the bank's expected return is 8% + 2% = 10%. The beta of the firm's debt is $2\% / 7\% = 0.286$.

Part A: The annual tax shields are $\$1{,}000 \times 8\% \times 35\% = \$28$ for the 8% coupon bond and $\$1{,}000 \times 6\% \times 35\% = \$21$ for the 6% coupon bond.

Question: This seems counterintuitive. The firm with the 6% bond is the higher rated firm, yet the firm with the 8% debt has the higher annual tax shields. The lower rated firm is more likely to default on its debt, and it would not get the tax shield, so it seems the higher rated firm, which is less likely to default, should have the

higher annual tax shields.

Answer: The higher rated firm has a lower pre-tax cost of debt as well as a lower after-tax cost of debt. The government takes a fixed percentage (35%) of the cost of debt. The higher rated firm's after-tax cost of debt is not as much lower as its pre-tax cost of debt; this is what we mean by the *lower rated firm's annual tax shields are larger*.

Part B: If the bonds are perpetuities, the present values of the annual tax shields are \$1,000 × 8% × 35% / 0.08 = \$350 for the 8% coupon bond and \$1,000 × 6% × 35% / 0.06 = \$350 for the 6% coupon bond. The present values of the tax shields are equal.

Question: Are you saying that a 6% coupon discounted at 6% is the same as an 8% coupon discounted at 8%? But 6% / 1.06 = 5.66% and 8% / 1.08 = 7.41%, so the present value of the coupon is greater for the lower rated firm. Since the tax shield is a fixed percentage of the coupon, the present value of the tax shield is also greater for the lower rated firm.

Answer: For debt of any *finite maturity*, the present value of the tax shield is greater for the lower rated firm.

Question: If it is true for debt of any finite maturity, isn't it true for perpetual debt as well? The present value of the second coupon payment is also greater for the lower rated firm, so the present value of the two coupon payments combined (proportional to the present value of the tax shield on a two year bond) is much greater for the lower rated firm.

Answer: But the difference gets smaller each period. One we get far enough out, the present value of the coupon is greater for the higher rated firm. We find this date as

$$6\% / 1.06^k = 8\% / 1.08^k \Rrightarrow$$
$$8\% \times 1.06^k = 6\% \times 1.08^k \Rrightarrow$$
$$ln\ 0.08 + k\ ln\ 1.06 = ln\ 0.06 + k\ ln\ 1.08 \Rrightarrow$$
$$k = (ln\ 0.08 - ln\ 0.06) / (ln\ 1.08 - ln\ 1.06) = 15.391$$

Question: Is there some simple intuition for this?

Answer: The market value of the debt is the same for the 6% debt and 8% debt, since both bonds were sold in an efficient market for the same par value. The present value of the return of principle is larger for the higher rated firm, since it has a lower capitalization rate.

- For finite debt, we infer that the present value of the coupon payments is higher for the lower rated firm.
- For perpetual debt, the present value of the return of principle is zero for both firms, so the present value of the coupon payments must be the same for both firms.

Part C: If the bonds have 1 year durations, the present values of the annual tax shields are \$1,000 × 8% × 35% / 1.08 = \$25.93 for the 8% coupon bond and \$1,000 × 6% × 35% / 1.06 = \$19.81 for the 6% coupon bond. The present value of the tax shields is larger for the greater coupon bond.

Exercise 17.4: Debt vs Equity

Firms A and B sell auto parts. Shares of each firm currently sell for $50, and both companies are financed predominantly by equity. The Chief Executive Officer (CEO) of Firm A feels that his stock is really worth $60, while the CEO of Firm B feels that his shares are really worth $40. Both CEOs would like to raise additional capital.

Which firm would prefer to issue debt and which firm would prefer to issue equity?

Solution 17.4:

Both firms sell their shares at $50, since that is the market price. Firm A's shares are selling at a discount, since the shares are actually worth $60 apiece. It would prefer to wait until the stock price rose to $60 and sell the shares at $60 apiece, so it issues debt, not equity.

Firm B's shares are selling at a premium, since it believes the shares are worth $40 apiece. It issues stock (equity) now, while the price is high, not debt.

Question: If the CEO of Firm A believes the shares are worth $60 apiece, why doesn't he buy the shares himself? And if the CEO of Firm B believes the shares are worth $40 apiece, why doesn't he sell short some shares himself?

Answer: Both CEO's would like to do this. But this is prohibited as insider trading.

Question: Don't Brealey and Myers say that capital markets are efficient, and a belief that the shares are worth more than their market price (or less than their market price) is likely to be mistaken?

Answer: That is true, unless the investor has more information than other investors. The CEO may know that the shares of the firm are truly worth more or less than their market price. As long as he doesn't trade for his own benefit (or that of his relatives or associates), he can issue shares or buy back shares of the company.

Question 17.5: Market Value of Firm

We have the following information about a firm.

- Price per share $10
- Outstanding shares 1,000,000
- Corporate tax rate 30%

The firm borrows $5,000,000 at an 8% yield and repurchases stock with the money. It intends to maintain its new debt-to-equity ratio. The debt issue increases the cost of financial distress by $500,000.

A. What is the value of the unlevered firm, before the issue of debt?
B. What is the present value of the tax shields from the debt?
C. What is the cost of financial distress?
D. What is the new market value of the firm, after the issue of debt?

Part A: The value of the unlevered firm is 1,000,000 shares at $10 a share = 1,000,000 × $10 = $10,000,000.

Part B: The value of the tax shield from perpetual debt of $5,000,000 at a 30% corporate tax rate is $5,000,000 × 30% = $1,500,000. The interest rate on the debt does not affect the solution if the debt is perpetual.

Part C: The additional cost of financial distress is $500,000.

Part D: The levered firm is $10,000,000 + $1,500,000 – $500,000 = $11,000,000.

Exercise 17.6: Return on Equity

A company has earnings before income and taxes (EBIT) of $11,560. The company has $100,000 of 6% debt outstanding. The corporate tax rate is 35%. The average equity during the year was $21,412.

A. What are the earnings after interest but before taxes?
B. What are the earnings after interest and taxes?
C. What is the return on *average* equity (ROE) for the company's stock?

Solution 17.6:

Part A: Interest is 6% × $100,000 = $6,000, so earnings after interest are $11,560 – $6,000 = $5,560.

Part B: Earnings after taxes are $5,560 × (1 – 35%) = $3,614.

Part C: The average equity during the year is $21,412, so the return on equity is $3,614 / $21,412 = 16.88%.