

Corporate Finance, Module 19: Adjusted Present Value

Debt Tax Shields Practice Problems

(The attached PDF file has better formatting.)

Question 19.1: Debt Yields and Maturities

A firm has a perpetual project, financed by a combination of debt and equity. The debt is fixed and is rolled over at maturity to the same principal; it is not re-balanced each year to the future value of the project. (If it were re-balanced, we would use the Miles-Ezzell formula, which is not covered on this course, but which may be tested on the CAS transition exam.) The corporate tax rate is 35%, and the personal tax rate is the same for equity income as for interest income. Under which of the following scenarios below would the value of the firm be highest? (Assume the costs of bankruptcy and the issue costs of debt are not material.)

- A. 10 year debt with a yield of 8% per annum
- B. 30 year debt with a yield of 8% per annum
- C. 10 year debt with a yield of 12% per annum
- D. 30 year debt with a yield of 12% per annum
- E. All of scenarios A, B, C, and D give the same value for the firm.

Answer 19.1: E or B; see below

Answer E: The project is perpetual, and the debt is rolled over when it matures, so *the debt is also perpetual*. (In general, Brealey and Myers assume that if the project is perpetual, the debt is perpetual.) Since the issue costs of debt are not material, the maturity of the current debt issue is not relevant. Since the debt is fixed (not re-balanced), the present value of the tax shields is the tax rate times the debt; it does *not* depend on the debt yield. Remember: the present value of the tax shields from perpetual fixed debt does not depend on the yield.

Question: This implies that the firm should not care what interest rate it must pay its creditors. If one bank offers an 8% loan and another bank offers a 12% loan, the firm is not indifferent; it prefers the loan with the lower interest rate. With the 12% loan, the firms pay out more cash each year to its creditors. The maturity of the loan is irrelevant only if the term structure of interest rates is flat. When the term structure is upward sloping (as is normally true), the firm expects to pay a higher yield on a longer term loan. If it can get both a 10 year loan and a 30 year loan at the same 8% rate, it prefers the 30 year loan. Shouldn't the answer be B?

Answer: You are correct; a financial analyst would pick B as the best way to finance debt. But if the loans are at the proper market rates and we consider only the present value of the tax shields, the options are the same. This type of question is ambiguous and would not be asked on the final exam; we include it here to focus on the tax shields.

Exercise 19.2: Present Value of Tax Shields

A firm has \$100 million face value of perpetual debt and a 35% marginal tax rate. The personal tax rate on interest income is the same as on equity income. The yield to maturity on the debt is 8% per annum. All rates in this exercise use annual compounding.

- A. If the coupon rate is 8%, what is the present value of the debt tax shields?
- B. If the coupon rate is 10%, what is the present value of the debt tax shields?

Solution 19.2:

Part A: If the coupon rate is 8%, the market value of the debt is \$100 million, and the present value of the perpetual tax shields is $\$100 \text{ million} \times 35\% = \35 million . Alternatively, the annual tax shield is $\$100 \text{ million} \times 8\% \times 35\% = \2.80 million , and the present value of a \$2.80 million perpetuity at an 8% discount rate is $\$2.80 / 8\% = \35 million .

Part B: If the coupon rate is 10%, the annual tax shield is $\$100 \text{ million} \times 10\% \times 35\% = \3.50 million . The present value of a \$3.50 million perpetuity at an 8% discount rate is $\$3.50 / 8\% = \43.75 million . Alternatively, the market value of the debt is $\$100 \text{ million} \times 10\% / 8\% = \125 million . The debt times the tax rate, or $\$125 \text{ million} \times 35\%$, is $\$43.75 \text{ million}$.

Question: We said above that the yield on the debt does not affect the present value of the tax shields. That seemed counter-intuitive, since the higher yielding bond has greater tax shields. Now you say that the 10% bond, which has a 25% higher yield than the 8% bond, has a present value of the tax shields that is 25% higher: $\$35 \text{ million} \times 1.25 = \43.75 million .

Answer: The yield on the two bonds is the same. They have the same par value and a perpetual maturity, so the 10% coupon bond has 25% higher market value and 25% higher present value of the tax shields.

The present value of the tax shields for a perpetual bond is the tax rate times the market value.

Exercise 19.3: Perpetual Debt

A firm has *perpetual* debt with a *face value* of \$50 million and *8% semi-annual coupons*. The *yield to maturity* on the debt is 6%, compounded semi-annually. The corporate tax rate is 35%, and the personal tax rate is the same for equity income as for interest income.

- A. What is the coupon payment each half year?
- B. What is the market value of this perpetual debt?
- C. What is the present value of the tax shields from the debt?

Solution 19.3:

Part A: The coupon payment each half year is $\$50 \text{ million} \times (\frac{1}{2} \times 8\%) = \2 million .

Part B: The market value of the perpetual debt is $\$2 \text{ million} / (\frac{1}{2} \times 6\%) = \66.67 million .

Part C: The present value of the tax shields is $\$66.67 \text{ million} \times 35\% = \23.33 million .

Alternatively, we reason as follows: if the coupon rate were 6%, the present value of the tax shields would be $\$50 \text{ million} \times 35\% = \17.50 million . Since the coupon rate is 8% instead of 6%, each coupon is $1\frac{1}{3}$ as large, so the present value of the tax shields is

$$\$50 \text{ million} \times 35\% \times 8\% / 6\% = \$23.33 \text{ million}.$$

Exercise 19.4: Interest Rate Changes

Debt with ten years remaining to maturity has a \$10 million par value, an 8% coupon, and a 6% yield to maturity (all rates use semi-annual compounding). The corporate tax rate is 35%.

- A. What is the coupon payment each half year?
- B. What is the tax shield each half year?
- C. What is the yield to maturity for a half year period?
- D. What is the present value of the tax shields?

Solution 19.4:

Part A: The coupon payment is $\frac{1}{2} \times 8\% \times \$10 \text{ million} = \$400,000$ each half year.

Part B: The tax shield is $\$10 \text{ million} \times \frac{1}{2} \times 8\% \times 35\% = \$140,000$ each half year.

Part C: The 6% yield to maturity is 3% each half year.

Part D: The present value of the semi-annual tax shields is $\$140,000 \times (1 - v^{20}) / 0.03$, where $v = 1/1.03 = 94.34\%$; this is \$2,082,846.

{The final exam does not test your knowledge of complex annuity formulas. This formula is in module 2 and is emphasized in the *readings* posting.