Corpfin modules 18 and 19: WACC and APV practice problems

Brealey and Myers, Chapter 19, “financing and valuation”

(The attached PDF file has better formatting.)

The textbook gives several methods to value projects and firms: WACC, APV, and the Miller and Modigliani adjustment to the opportunity cost of capital (r_{MM}). The methods are confusing at first, because they are used for different scenarios and some results may seem counter-intuitive.

The WACC method includes the effects of debt tax shields in the weighted average cost of capital. The APV method uses the opportunity cost of capital for the base case NPV and separately quantifies the financing side effects. Brealey and Myers mention three such side effects:

- Interest tax shields on the debt supported by the project increase the APV of the project
- Issue costs of securities financing the project decrease the APV of the project
- Financing provided by a supplier at below market interest rates increase the APV of the project

Question: Why do we use the opportunity cost of capital for the base case NPV instead of the WACC?

Answer: The base case NPV assumes no tax shields from debt. The tax shields are separately quantified and added to the APV.

Question: What capitalization rate is used for the present value of financing side effects?

Answer: For the tax shields of debt, use the borrowing rate if the debt is fixed and the opportunity cost of capital is the debt is continually re-balanced to a percentage of the project’s market value. For issue costs of securities, no capitalization rate is needed, since the costs are up-front. For subsidized borrowing, use the borrowing rate of the firm.

Know the types of scenarios: perfect capital markets vs actual capital markets, perpetual debt vs debt with limited periods, and fixed debt vs re-balanced debt.

The exercises on the discussion forum cover the items asked on final exam problems. Many exercises ask for the computation of NPV’s, returns on capital, or betas, or explanation of why the values, returns, or betas increase or decrease as the input parameters change. The solutions provide clear explanation; the dialogues add intuitive commentary and recommendations for optimal exam preparation. The final exam problems are multiple choice, to ensure objective grading.

Adjusted present value (APV) is a more flexible method to value a project than WACC in two ways.

First, the WACC method assumes the debt is perpetual. The APV method can be used for limited debt terms as well. The present value of a perpetual cash flow $Z is $Z / R%, where R% is the appropriate discount rate. For cash flows over a limited period, take the sum of the present values of each flow. If the final exam problem gives a one year or two year project, use the APV method. The discussion forum shows a sample one year project valued by the APV method.

The weighted average cost of capital (WACC) method is used only when the debt-to-equity ratio is constant. As the market value of the project changes, the amount of debt changes to keep the debt-to-equity ratio the same. At first glance, this seems unrealistic, as though the firm increases or decreases the face value of the debt whenever the market value of the project rises or falls. The transaction costs of changing the face value of the debt are too high. Re-balancing the debt means that if the value of the project doubles next year because demand for the firm’s products doubles and a new factory must be built, the firm issues more debt. Similarly, if the project’s value decreases over the next 30 years as the firm’s product becomes obsolete, the debt declines accordingly. These are realistic assumptions.

Final exam problems on the APV method state if the debt is fixed vs re-balanced. If the exam problem asks for the weighted average cost of capital (WACC), assume the debt is re-balanced continually to a constant debt-to-equity ratio. If the exam problem gives the debt-to-equity ratio (or if it gives the debt-to-value ratio), assume the debt is re-balanced continually to this ratio. If the final exam problem says the debt has a limited duration, assume the debt is fixed at the initial face value.

Focus first on the WACC method for perfect capital markets. Know how to derive the returns on debt capital, equity capital, and assets, and the associated betas. You use the formulas even for actual capital markets to derive the return on equity capital and the WACC. The textbook gives a three step procedure to determine the weighted average cost of capital (WACC) in actual capital markets with a positive corporate tax rate. Exam problems on the WACC often use the three step procedure.

Some results for actual capital markets seem counter-intuitive. Brealey and Myers explain the intuition several ways, since some results appear strange. For example, if the opportunity cost of capital stays the same but the borrowing cost increases because the risk of bankruptcy increases, one might suppose that the value of the firm decreases. After all, it pays more to bondholders for its capital, so its value should decrease. But Brealey and Myers conclude the opposite: the value of the firm increases. The exercises on the discussion forum and the Jacob - Rachel dialogues explain why.

The APV method uses the opportunity cost of capital for the base case NPV. The final exam problem may give the opportunity cost of capital or data to derive the opportunity cost of capital. The data are normally the risk-free rate, the market risk premium, and the beta of the project.

** Exercise 19.1: Adjusted present value

A project with an initial investment for the project is $2.5 million has perpetual cash inflows of $0.225 million per annum beginning in one year.

- The opportunity cost of capital for the project is 12%.
- The project supports borrowing of $1 million.
- The marginal tax rate is 35%.
- The borrowing cost for $1 million of debt is 6% per annum.

A. With all equity financing, what is the net present value of the project?
B. If the debt is continually re-balanced, what is the present value of the interest tax shields supported by the project?
C. If the debt is continually re-balanced, what is the APV of the project?
D. If the debt is fixed, what is the present value of the interest tax shields supported by the project?
E. If the debt is fixed, what is the APV of the project?

Part A: The opportunity cost of capital is the discount rate with all equity financing. The NPV of the project is

Base case NPV = –2.5 million + 0.225 million / 0.12 = $-0.625 million

Part B: The annual interest tax shield is 35% × 6% × 40% × $2.5 million = $21,000.

If the debt is continually re-balanced, use the opportunity cost of capital as the discount rate:

$21,000 / 12% = $175 thousand.

Part C: The APV of the project is –$625,000 + $175,000 = $-450 thousand

Part D: If the debt is fixed, use the firm's borrowing cost as the discount rate:

$21,000 / 6% = $350 thousand.

Part E: The APV of the project is –$625,000 + $175,000 = $-275 thousand

Question: This seems counter-intuitive. If the debt is fixed, the firm has more risk, since it must pay the debt costs even if the project is not profitable. If the firm has more risk, why is the NPV of the project higher?

Answer: Brealey and Myers discuss this briefly; not all economists agree with their interpretation. In the APV method, debt affects the value of the project by its tax shields. More debt also makes the project more risky, but it adds more tax shields and so increases the value of the project.

Question: Adding more debt increases the value of the project intuitively, since the cost of debt capital is lower. But the comparison of fixed vs re-balanced debt seems strange. If the debt is fixed, the firm has more risk, since it must pay the debt costs even if the project is not profitable. If the firm has more risk, why is the NPV of the project higher?

Answer: Brealey and Myers mention that if the debt is fixed, the firm *may have* more risk, since it must pay the debt costs even if the project is not profitable. They do not say that the firm is definitely more risky. If the firm re-balances the debt, it repays principal when it does poorly (has less income) and increases the principal when it does well (has more income). This raises risk: having to repay principal when income is low raises the likelihood of bankruptcy.

[The exercise below is the illustration in the textbook. Be sure you understand how the pieces fit together. The final exam problems are similar, asking for the adjusted present value of the project in two scenarios: debt fixed at a dollar amount vs debt re-balanced each year to a percentage of the project's market value.]

Exercise 19.2: Adjusted present value

A project with an initial investment for the project is $12.5 million has perpetual cash inflows of $1.125 million per annum beginning in one year.

- The opportunity cost of capital for the project is 9.84%.
- The project supports borrowing of $5 million (40% of its value).
- The marginal tax rate is 35%.
- The borrowing cost for $5 million of debt is 6% per annum.

A. With all equity financing, what is the net present value of the project?
B. If the debt is continually re-balanced, what is the WACC of the project?
C. What is the NPV of the project with the 40% debt using the WACC?
D. If the debt is continually re-balanced, what is the present value of the debt interest tax shields supported by the project?
E. If the debt is continually re-balanced, what is the APV of the project?
F. If the debt is fixed, what is the present value of the debt interest tax shields supported by the project?
G. If the debt is fixed, what is the APV of the project?

Part A: The opportunity cost of capital is the discount rate with all equity financing. The NPV of the project is

Base case NPV = –12.5 million + 1.125 million / 0.0984 = $-1.067 million

Part B: The weighted average cost of capital is determined in two steps.

Step #1: Determine the cost of equity capital at a 40% debt-to-value ratio by the Miller and Modigliani equation:

40% × 6% + 60% × Z = 9.84%
⇛ Z = (9.84% – 40% × 6%) / 60% = 12.40%

Step #2: Determine the weighted average cost of capital at a 40% debt-to-value ratio:

WACC = 40% × 6% × (1 – 35%) + 60% × 12.40% = 9.00%

The WACC method is used only if the debt is continually re-balanced. If the debt is fixed, the present value of the debt tax shields discounts at the borrowing rate, so the present value is higher and the WACC gives too *low* a value.

Part C: The NPV of the project is

NPV = –12.5 million + 1.125 million / 0.09 = $0.000 million

Part D: The annual interest tax shield is 35% × 6% × 40% × $12.5 million = $105,000.

If the debt is continually re-balanced, use the opportunity cost of capital as the discount rate:

$105,000 / 9.84% = $1,067 thousand.

This is the negative of the base case NPV.

Part E: The APV of the project is –$1,067,000 + $1,067,000 = zero.

Part F: If the debt is fixed, use the firm's borrowing cost as the discount rate:

$105,000 / 6% = $1,750 thousand.

Part G: The APV of the project is –$1,067,000 + $1,570,000 = $683,000.

Question: This seems counter-intuitive. If the debt is fixed, the firm has more risk, since it must pay the debt costs even if the project is not profitable. If the firm has more risk, why is the NPV of the project higher?

Answer: Brealey and Myers discuss this briefly; not all economists agree with their interpretation. In the APV method, debt affects the value of the project by its tax shields. More debt also makes the project more risky, but it adds more tax shields and so increases the value of the project.

Question: Adding more debt increases the value of the project intuitively, since the cost of debt capital is lower. But the comparison of fixed vs re-balanced debt seems strange. If the debt is fixed, the firm has more risk, since it must pay the debt costs even if the project is not profitable. If the firm has more risk, why is the NPV of the project higher?

Answer: Brealey and Myers mention that if the debt is fixed, the firm *may have* more risk, since it must pay the debt costs even if the project is not profitable. They do not say that the firm is definitely more risky. If the firm re-balances the debt, it repays principal when it does poorly (has less income) and increases the principal when it does well (has more income). This raises risk: having to repay principal when income is low raises the likelihood of bankruptcy.

** Exercise 19.3: Adjusted present value

A project with an initial investment of $100 million has perpetual cash inflows of $20 million per annum.

- The opportunity cost of capital for the project is 12%.
- The project supports borrowing of $20 million (fixed) at a 6% cost of debt capital.
- The marginal tax rate is 35%.

If the cost of debt capital is 7% instead of 6%, is the value of the project higher or lower?

Question: This seems obvious. If the firm pays 1% more interest, it has less cash left, so the value of the project is lower.

Answer: Your reasoning is the pre-Miller and Modigliani logic. In perfect capital markets, the capital structure does not affect the value of the project. With all-equity financing, the cost of debt capital does not matter.

Question: Explain the logic for no change in the value of the project if the cost of debt capital increases.

Answer: If the cost of debt capital increases but the opportunity cost of capital does not change, this may mean that the risk-free rate increases but the systematic risk of the project decreases or that the probability of bankruptcy increases. In both cases, the higher value to shareholders offsets the lower value to bondholders.

Question: Are you saying that the change in the cost of debt capital does not change the APV of the project?

Answer: If the debt is perpetual, the present value of the debt tax shields is the tax rate T_c × the market value of the debt. The cost of debt capital does not affect the present value of the debt tax shields.

In other scenarios, the change in the cost of debt capital affects the present value of the debt tax shields. If the yield to maturity of the debt increases (perhaps because the risk-free rate increases), the market value of the debt decreases, so the present value of the debt tax shields decreases. This would raise the APV of the project.

Question: That is counter-intuitive. If the firms more for its debt, the present value of the project should decrease.

Answer: You are correct; if the cost of debt capital increases because the risk-free rate increases, one would expect the opportunity cost of capital to increase. The assumption in this exercise is that the opportunity cost of capital stays the same. The intuition has two parts.

- In perfect capital markets, the value of the project is the same for all debt-to-equity ratios. The opportunity cost of capital has not changed, so the value of the project with all-equity financing does not change. The cost of debt capital does not affect the value of the project.
- In actual capital markets, the firm gains from the present value of the debt tax shields. The higher cost of debt capital raises the present value of the debt tax shields *if the debt is continually re-balanced*.

** Exercise 19.4: Borrowing rates and NPV

- An insurer invests $25 million to build an agency force (buying offices, hiring agents from competitors).
- It expects perpetual income of $2.25 million a year.
- Its opportunity cost of capital is 10% per annum (with all equity financing).
- It can borrow $10 million to support the project (building the agency force).

Its borrowing rate is 4%, 6%, or 8%.

A. With which borrowing rate is the net present value of the project greatest in perfect capital markets?
B. With which borrowing rate is the NPV of the project greatest if the corporate tax rate is positive?

Part A: With perfect capital markets, the capital structure does not affect the value of the project. With all-equity financing, the value of the project is –$25 million + $2.25 million / 10% = $-2.50 million.

Part B: If the debt is fixed at $10 million, the present value of the debt tax shields is $10 million × T_c (the corporate tax rate). This present value does not depend on the borrowing rate. If T_c = 35%, the present value of the project is –$2.5 million + $3.5 million = $1 million.

If the debt is rebalanced each year to the same percentage of the project's market value, the present value of the debt tax shields is $10 million × T_c × r_D / 10%. This value is greatest for the 8% borrowing rate and smallest for the 4% borrowing rate.

Question: I would expect the opposite. With a 4% borrowing rate, the firm pays half the interest expense as with an 8% borrowing rate. The interest costs are subtracted from the NPV of the project, so the NPV should be highest with the 4% interest rate.

Answer: The adjusted present value of the project is the base case NPV with all-equity financing plus the present value of the debt tax shields. Know the principles.

- If the debt is fixed, the present value of the debt tax shields does not depend on the borrowing rate.
- If the debt is re-balanced each year, the present value of the debt tax shields increases with the borrowing rate.

Question: How can the NPV of the project increase if its costs increase?

Answer: In perfect capital markets, the NPV of the project does not depend on its financing. If the debt rate is higher, the equity rate must be lower to yield the same overall capitalization rate. With taxes, the government pays part of the debt cost. A higher debt cost means the government pays more, so the NPV of the project rises.

Question: You say that if the debt cost increases, the equity cost must be lower. A higher debt costs means the project is riskier. Doesn't that mean the equity cost is higher?

Answer: A higher debt cost means there is a greater chance the firm will default. If it defaults, the bondholders suffer the loss. Given the overall risk of the project, a higher probability of default means the firm will not pay its debt to bondholders. The shareholders walk away from the firm's debt, so they gain from default.

Question: If that is true, shouldn't stockholders gain if the firm has more debt? Why do Miller and Modigliani say the firm's value does not depend on its debt ratio in perfect capital markets?

Answer: The bondholders set the debt rate to cover their expected losses from defaults. If the default rate is higher, they charge more for the debt. The higher cost of debt offsets the stockholders' gain from default.

** Exercise 19.5: Leveraged buyouts

Leveraged buyouts (LBO's) are takeovers, typically of mature companies, financed almost entirely with debt. The new debt is not intended to be permanent. LBO business plans call for generating extra cash by selling assets, shaving costs, and improving profit margins. The extra cash is used to pay down the LBO debt.

A. Are leveraged buyouts valued by the WACC method or the APV method? Explain why.
B. What is the rationale for leveraged buyouts?

Part A: The WACC method can be used only when the debt is continually re-balanced to a constant debt-to-equity ratio. The debt in a leveraged buyout is reduced (by selling assets, shaving costs, and improving profit margins), so one can't use the WACC method. Use the APV method.

Part B: The textbook does not explain the rationale for leveraged buyouts.

Question: The textbook says these are mature companies that are refinanced almost entirely with debt. Is the rationale the debt tax shields?

Answer: Senior managers of mature companies are often non-aggressive. The industries are mature, with few new products or new competitors. Examples are hotels, trucking, railroads, steel, cement, ocean shipping. Managers know their counterparts in other companies. They know that if one firm tries to gain market share by cutting prices, other firms follow suit and cut prices as well, and everyone loses. So all firms prefer to maintain prices and products with limited competition. No one competes too hard, and everyone is happy.

[Note: The rationale given by Rachel is not in the textbook and is not tested on the final exam.]

** Exercise 19.6: WACC

Suppose WACC is estimated as follows at a 30% debt to value ratio as

WACC = r_D (1 – T_c) (D/V) + r_E (E/V) = 0.09 (1 – 0.35)(0.30) + 0.15(0.70) = 0.1226 or 12.26%

What is the WACC at a 50% debt to value ratio if the cost of debt capital r_D at a 50% debt ratio is 9.5%?

Solution 19.6: Use the three step procedure in the Brealey and Myers textbook:

Step #1: Calculate the opportunity cost of capital.

r = r_D (D/V) + r_E (E/V) = 0.09 (0.30) + 0.15 (0.70) = 0.132 or 13.2%

Step #2: Calculate the new costs of debt and equity.

The cost of debt r_D at a 50% debt is 0.095. The new cost of equity is

r_E = r + (r – r_D) (D/E)
= 0.132 + (0.132 – 0.095) (50/50)
= 0.169 or 16.9%

Step #3: Calculate the WACC at a 50% debt to value ratio:

WACC = r_D (1 – T_c) (D/V) + r_E (E/V)
= 0.095 (1 – 0.35) (0.50) + 0.169 (0.50) = 0.1154 or 11.54%.

** Exercise 19.7: WACC

- Debt of $75 million (book value) is outstanding.
- The debt is trading at 90% of book value.
- The yield to maturity is 9%.

- 2.5 million shares are selling at $42 per share.
- The expected rate of return on the stock is 18%.

The marginal tax rate is 35%.

The debt is re-balanced to the same debt-to-equity ratio each year.

A. What are the market values of debt and equity?
B. What are the debt to value and equity to value ratios?
C. What is the weighted average cost of capital (WACC)?

Part A: We use market values, not book values.

- The market value of debt is $75 million × 90% = $67.5 million.
- The market value of equity is 2.5 million shares × $42 a share = $105.0 million.

Part B: The WACC weights the cost of debt capital and the cost of equity capital.

- The debt to value ratio is $67.5 / ($67.5 + $105.0) = 39.1304%.
- The equity to value ratio is $105 / ($105 + $105.0) = 60.8696%.

Part C: The weighted average cost of capital is

39.1304% × 9% × (1 – 35%) + 60.8696% × 18% = 13.25%

Question: This problem gives the yield to maturity on the bonds. What if the problem gives the coupon rate?

Answer: An exam problem using the APV (adjusted present value) method may give a coupon rate on a bond. From the coupon rate, determine the interest paid and the tax shield on the interest. Problems on WACC give the yield to maturity, not the coupon rate.

** Exercise 19.8: Balance sheet and WACC

A firm's interest rates are 9% on long-term debt and 10% on short-term bank financing. The market value of the debt is the same as it book value.

The firm has 10 million shares outstanding, trading at $90 per share. The expected return on the common stock is 18% per annum.

The marginal tax rate is 35%.

A. What is the firm's WACC?
B. If the firm pays a one-time $10 dividend per share, what is the new WACC? Assume the cost of debt capital r_D does not change.

Cash + marketable securities	100		Bank loan	280
Accounts receivable	200		Accounts payable	120
Inventory	50		Current Liabilities	400
Current assets	350			
Real estate	2,100		Long-term debt	1,800
Other assets	150		Equity	400
Total assets	2,600		Total liabilities + equity	2,600

Part A: The market value of equity is $90 × $1 million = $900 million. The WACC is

(280 × 10% × (1 – 35%) + 1,800 × 9% × (1 – 35%) + 900 × 18%) / (280 + 1,800 + 900) = 9.58%.

Part B: The dividend causes the stock price to fall $10 to $80 a share. The WACC is

(280 × 10% × (1 – 35%) + 1,800 × 9% × (1 – 35%) + 800 × 18%) / (280 + 1,800 + 800) = 9.29%.

[Final exam problems may give balance sheet entries, from which the weighted average cost of capital, firm value, or returns on assets or equity may be computed.]

** Exercise 19.9: WACC

The debt in the balance sheet below (figures are in thousands of dollars) has just been refinanced at an interest rate of 6% (short term) and 8% (long term). The expected rate of return on the firm's shares is 15%. 7.46 million shares are outstanding, and the shares trade for $46. The marginal tax rate is 35%.

The firm issues $50 million in new equity and uses the proceeds to retire long-term debt. The firm's borrowing rates do not change.

A. What is the firm's weighted average cost of capital before the refinancing?
B. What is the cost of equity capital after the refinancing?
C. What is the firm's weighted average cost of capital after the refinancing?

	($000)		($000)
Cash + marketable securities	1,500	Bank loan	75,600
Accounts receivable	120,000	Accounts payable	62,000
Inventory	125,000	Current Liabilities	137,600
Current assets	246,500	Deferred tax liabilities	45,000
Property, plant, equipment	302,000	Long-term debt	208,600
Other assets	89,000	Shareholders' equity	246,300
Total assets	637,500	Total liabilities + equity	637,500

Part A: The market value of the equity is 7.46 million × $46 = $343,160,000

Part B: The weighted average cost of capital is

($75,600 × 6% × (1 – 35%) + $208,600 × 8% × (1 – 35%) + $343,160 × 15%) / ($75,600 + $208,600 + $343,160) = 10.40%

** Exercise 19.10: Two year project

A projects initial investment is $1 million at time t=0 and expected after-tax returns are $600,000 at time t=1 and $700,000 at time t=2. The project lasts for two years only.

The opportunity cost of capital is 12% with all-equity financing, the borrowing rate is 8%, and the firm borrows $300,000 against the project.

- The debt will be repaid in two equal installments.
- The corporate tax rate is 35%.

A. What is the base case NPV?
B. What is the interest paid in each year?
C. What is the present value of the debt tax shields?
D. What is the value of the project?

Part A: The base case NPV is –$1 million + $600,000 / 1.12 + $700,000 / 1.12^2 = $93,750.

Part B: The interest paid in the first year $300,000 × 8% = $24,000. Half the debt is repaid at the end of the first year, and the principal remaining at the beginning of the second year is $150,000. The interest paid in the second year $150,000 × 8% = $12,000.

Part C: The present value of the interest tax shields is

$24,000 × 30% / 1.08 + $12,000 × 30% / 1.08^2 = $9,753.

Part D: The value of the project is $93,750 + $9,753 = $103,503.

Exercise 19.11: Adjusted present value

A project's initial investment is $1 million, and the expected annual cash inflow starting one year after the initial investment is $95,000 in perpetuity. The opportunity cost of capital with all-equity financing is 10%, and the project allows the firm to borrow at 7%. The marginal tax rate is 35%.

A. What is the base case NPV of the project with all-equity financing?
B. What is the present value of tax shields if $400,000 of fixed, perpetual debt is used?
C. What is the project's value if $400,000 of fixed, perpetual debt is used?
D. What is the present value of tax shields if the debt is increased or decreased in proportion to changes in the market value of the project?
E. What is the project's value if the debt is increased or decreased in proportion to changes in the market value of the project?

Part A: The base case NPV of the project with all-equity financing is

–$1 million + $95,000 / 10% = $-50,000

Part B: The present value of tax shields if $400,000 of fixed, perpetual debt is used is

35% × 7% × $400,000 / 7% = $140,000

Part C: The project's value if $400,000 of fixed, perpetual debt is used is

–$50,000 + $140,000 = $90,000.

Part D: The present value of tax shields if the debt is increased or decreased in proportion to changes in the market value of the project is

35% × 7% × $400,000 / 10% = $98,000

Part E: The project's value if the debt is increased or decreased in proportion to changes in the market value of the project is

–$50,000 + $98,000 = $48,000.

** Exercise 19.12: Adjusted present value

A project's initial investment is $1 million, and the expected annual cash inflow starting one year after the initial investment is $95,000 in perpetuity. The opportunity cost of capital with all-equity financing is 10%, and the project allows the firm to borrow at 7%. The marginal tax rate is 35%.

A. What is the base case NPV of the project with all-equity financing?
B. What is the present value of tax shields if $400,000 of fixed, perpetual debt is used?
C. What is the project's value if $400,000 of fixed, perpetual debt is used?
D. What is the present value of tax shields if the debt is increased or decreased in proportion to changes in the market value of the project?
E. What is the project's value if the debt is increased or decreased in proportion to changes in the market value of the project?

Part A: The base case NPV of the project with all-equity financing is

–$1 million + $95,000 / 10% = $-50,000

Part B: The present value of tax shields if $400,000 of fixed, perpetual debt is used is

35% × 7% × $400,000 / 7% = $140,000

Part C: The project's value if $400,000 of fixed, perpetual debt is used is

–$50,000 + $140,000 = $90,000.

Part D: The present value of tax shields if the debt is increased or decreased in proportion to changes in the market value of the project is

35% × 7% × $400,000 / 10% = $98,000

Part E: The project's value if the debt is increased or decreased in proportion to changes in the market value of the project is

–$50,000 + $98,000 = $48,000.

** Exercise 19.13: Refinancing

A firm's cost of debt capital r_D depends on its debt-to-value ratio D/V as $r_D = 5\% \times (1 + D/V)$.
The corporate tax rate $T_c = 35\%$.

At a 20% debt-to-value ratio, the firm's WACC is 10%.

A. At a 20% debt-to-value ratio, what is the firm's cost of debt capital r_D?
B. At a 20% debt-to-value ratio, what is the firm's cost of equity capital r_E?
C. What is the firm's opportunity cost of capital *r*?
D. At a 40% debt-to-value ratio, what is the firm's cost of debt capital r_D?
E. At a 40% debt-to-value ratio, what is the firm's cost of equity capital r_E?
F. At a 40% debt-to-value ratio, what is firm's WACC?

Part A: At a 20% debt-to-value ratio, the firm's cost of debt capital r_D is $5\% \times (1.2) = 6\%$.

Part B: At a 20% debt-to-value ratio, $20\% \times 6\% \times (1 - 35\%) + 80\% \times r_E = 10\% \Rightarrow$

$r_E = [\ 10\% - 20\% \times 6\% \times (1 - 35\%)\ ] / 80\% = 11.5250\%$

Part C: The firm's opportunity cost of capital *r* is $20\% \times 6\% + 80\% \times 11.525\% = 10.4200\%$

Part D: At a 40% debt-to-value ratio, the firm's cost of debt capital r_D is $5\% \times (1.4) = 7\%$.

Part E: At a 40% debt-to-value ratio, $40\% \times 7\% \times + 60\% \times r_E = 10.42\% \Rightarrow$

$r_E = [\ 10.42\% - 40\% \times 7\%\ ] / 60\% = 12.7000\%$

Part F: At a 40% debt-to-value ratio, the firm's WACC is

$40\% \times 7\% \times (1 - 35\%) + 60\% \times 12.7\% = 9.4400\%$

** Exercise 19.14: Risk, return on capital, and value

A firm has cash inflows of $24 million per annum in perpetuity.

A. If the firm's systematic risk increases, does its cost of equity capital increase or decrease? Does its value increase or decrease?
B. If the firm's debt-to-equity ratio increases, does it risk of default increase or decrease? Does its cost of equity capital increase or decrease? Does its value increase or decrease?
C. Reconcile these two statements.

Part A: As systematic risk increases, the opportunity cost of capital increases and the required return on equity increases. If the firm's expected cash flows do not change, its value decreases.

Question: Are you saying that more risky firms have lower values? If so, why would firms take risks?

Answer: This exercise assumes the expected cash flows do not change. More risky firms generally have higher expected cash flows.

Part B: As the firm's debt-to-equity ratio increases, its risk of default increases.

- Its cost of debt capital increases and its cost of equity capital increases.
- In perfect capital markets, its value does not change.
- If the corporate tax rate is more than zero and the costs of bankruptcy are not material, the firm's return on assets decreases and its value increases.

Part C: The firm's value depends on its systematic risk and its tax shields.

- As systematic risk increases and the expected cash inflows do not change, the firm's value decreases.
- As the firm's tax shields increase, the firm's value increases.

Tax shields imply that the government pays part of the firm's liabilities. The more that the government pays, the greater the value of the firm.

Question: What happens to the value of the equity as the debt-to-equity ratio increases?

Answer: The firm's value increases, the value of debt decreases or remains level, so the value of equity increases.

Question: A higher risk of bankruptcy is a higher probability that shareholders will lose their investment. The bondholders precede shareholders in a bankruptcy: shareholders get paid only after bondholders receive all their claims. If bondholders lose by a higher probability of bankruptcy, surely shareholders lose.

Answer: Bondholders receive specified interest payments if the firm is solvent and nothing if the firm becomes bankrupt. A higher probability of bankruptcy means a lower expected cash inflow, so bondholders lose.

Stockholders receive the value of the firm after the bondholders receive their coupons. With more debt, the financial leverage of the firm increases. Since the cost of debt capital is less than the cost of equity capital, the expected return to stockholders increases.

- In good years, the stockholders receive much more.
- In bad years, the stockholders receive less.

The expected return to stockholders in *perfect capital markets* does not change. The systematic risk of equity increases with the expected return, and the value of equity does not change.

In actual capital markets, stockholders benefits two ways.

- Stockholders get the present value of the debt tax shields. The debt tax shields benefit the firm paying the interest, not the bondholders receiving the interest. The stockholders own the firm, so they benefit.
- Stockholders they gain from limited liability. In good years, stockholders' potential gain is unlimited. In bad years, stockholders' maximum loss is their initial investment. Higher probability of bankruptcy may benefit stockholders if it is not accompanied by higher costs of bankruptcy.

[Brealey and Myers get the same results from different methods, but readers often have trouble replicating their work. This exercise explains what they do and points out the confusing parts.]

** Exercise 19.15: Perpetual cash flows and valuation methods

A project with an initial investment of $3 million is expected to have perpetual cash inflows of $360,000 a year, starting one year after the initial investment.

The opportunity cost of capital for projects of similar risk is 12% per annum.

- The project supports debt equal to one third of its value.
- It is financed one third debt and two thirds equity.
- The required return on the debt is 6% per annum.

The corporate tax rate is 35%.

A. Assume the debt is rebalanced each year to one third of the project's value. Use the WACC method to compute the net present value of the project.
B. Assume the debt is rebalanced each year to one third of the project's value. Use the APV method to compute the net present value of the project.
C. Assume the debt is fixed at $1 million. Use the APV method to compute the net present value of the project.
D. Assume the debt is fixed at $1 million. Use the Miller and Modigliani method to compute the net present value of the project.
E. Assume the issue cost of stock is 15% of the net proceeds and the issue cost of debt is 2% of the net proceeds. For each assumption about the debt (fixed vs rebalanced), what is the NPV of the project?

Part A: Use the three step method in the textbook.

Find the cost of equity capital r*E*: ⅓ × 6% + ⅔ × rE = 12% ⇛ rE = (12% – 2%) × 1.5 = 15.00%.

Find the weighted average cost of capital: ⅓ × 6% × (1 – 35%) + ⅔ × 15% = 11.3000%.

Find the NPV of the project: –$3 million + $360,000 / 11.30% = $185,841

Part B: The base case NPV is –$3 million + $360,000 / 0.12 = $0.

The debt is one third of $360,000 / 11.3% = $3,185,841.

The present value of the debt tax shields is 6% × 35% × ⅓ × $3,185,841 / 0.12 = $185,841.

The NPV of the project is $0 + $185,841 = $184,841.

Question: This solution seems contrived. From the base case in the APV method, the value of the project seems to be $360,000 / 12% = $3 million. The present value of the debt tax shields is 6% × 35% × ⅓ × $3 million / 0.12 = $175,000. The NPV of the project is $0 + $175,000 = $175,000. The APV method comes to the same result as the WACC method only because we use the WACC of 11.3% to derive the value of the project, the amount of debt, and the present value of the debt tax shields.

Answer: You are right. The textbook avoids these complexities by contrived examples. Your comment should help candidates with final exam problems. Be sure to use the debt to value ratio times the market value of the project. Some final exam problems say that the debt is $1 million and is re-balanced each year. This type of problem is best solved by the APV method. The WACC method is harder because you don't know the market value of the project so you don't know the debt-to-equity ratio. The principles are:

- If the problem gives a debt to value ratio and says the debt is re-balanced each year, use the WACC method. If the problem just gives a debt to value ratio, assume the debt is re-balanced each year.
- If the problem gives a debt amount and says the debt is re-balanced each year, use the APV method with the opportunity cost of capital to derive the present value of the debt tax shields.
- If the problem gives a debt amount and says the debt is fixed, use the APV method with the borrowing rate to derive the present value of the debt tax shields.

Part C: The base case NPV is –$3 million + $360,000 / 1.12 = $0.

The present value of the debt tax shield is 6% × 35% × ⅓ × $3 million / 0.06 = $350,000

Part D: The value of the project is the present value of future cash flows. From the APV method above, thsi value is $3,350,000. The discount rate for the Miller and Modigliani method is $r_{MM} = r \times (1 - T_c D/V)$

$$= 12\% \times (1 - 35\% \times \$1 \text{ million} / \$3.35 \text{ million}) = 10.74627\%$$

The NPV of the project is –$3 million + $360,000 / 0.1074627 = $350,000

Question: What is the use of the Miller and Modigliani method? We calculated the value of the project from the APV method to get D / V, which gives the discount rate for the Miller and Modigliani method, which gives the value of the project? Isn't this circular?

Answer: This explains why Brealey and Myers just mention the Miller and Modigliani formula, without much discussion. We don't use it to independently calculate the net present value of projects.

Part E: The issue costs are 15% × $2 million + 2% × $1 million = $320,000.

If the debt is rebalanced as the projects value changes, the NPV is $175,000 – $320,000 = $-145,000

If the debt is fixed, the NPV is $350,000 – $320,000 = $30,000

** Exercise 19.16: One year project

A firm undertakes a one year project with the following attributes.

- The initial investment is $1 million.
- The project provides an expected return of $1.1 million at the end of the year.

- The opportunity cost of capital is 12%
- The return on debt r_D is 8%
- The debt to value ratio (D/V) is 20%

The corporate tax rate is 35%.

A. What is the base case NPV?
B. What is the present value of the debt tax shield?
C. What is the adjusted present value of the project?

Part A: The base case NPV is –$1 million + $1.1 million / 1.12 = $-17,857

Part B: The present value of the debt tax shield is

35% × 20% × 8% × $1 million / 1.08 = $5,185

Part C: –$17,857 + $5,185 = $-12,672