

Corporate Finance, Module 18: Weighted Average Cost of Capital

Practice Problems

(The attached PDF file has better formatting.)

Question 18.1: WACC

A firm has debt of \$10 million valued at 100% of par and paying 8% interest. The firm has 5 million shares outstanding, selling at \$5 per share. The company expects to earn \$5 million *before interest and taxes* each year in perpetuity. The marginal tax rate is 35%.

- What is the after-tax cost of debt capital? (The pre-tax cost times the complement of the corporate tax rate.)
- What is the earnings of the firm after interest payments but before tax payments? (Subtract the par value of the debt times the coupon rate from the earnings before interest and taxes.)
- What is the after-tax earnings of the firm available to shareholders? (Earnings after interest payments times the complement of the corporate tax rate.)
- What is the market value of the equity? (Number of share times market price per share.)
- What is the return on (market value) equity (r_e)? (After-tax earnings to shareholders divided by market value of equity.)
- What is the return on assets? Use the weighted average cost of capital method. (Weight the after-tax cost of debt capital with the cost of equity capital, where the market value of debt and equity are the weights.)

Solution 18.1:

Part A: The debt is \$10 million. The pre-tax cost of debt capital is 8% per annum. The after-tax cost of debt capital is $8\% \times (1 - 35\%) = 5.2\%$.

Question: The coupon payments on corporation debt are tax deductible. If the firm has \$10,000 of pre-tax coupon payments, its pre-tax wealth decreases by \$10,000 and its after-tax wealth decreases by \$10,000. Why is the after-tax cost of debt capital different from the pre-tax cost of debt capital?

Answer: The terms pre-tax and after-tax are confusing here. Suppose a firm has a project that returns 10% on the capital invested after paying all costs, including federal income taxes. The firm needs \$100,000 of capital for the project. The project returns a pre-tax profit of \$15,385 each year. It pays federal income taxes of $\$15,385 \times 35\% = \$5,385$, and has an after-tax profit of $\$15,385 \times (1 - 35\%) = \$10,000$.

The firm can obtain the \$100,000 of required capital either from shareholders or creditors, both of whom require a 12% return, or \$12,000 each year. The providers of capital, whether shareholders or creditors, are not concerned with the firm's tax payments; they simply want \$12,000 a year.

- If it takes the money from shareholders, it must give them \$12,000 each year from its after-tax earnings. It has only \$10,000 in after-tax earnings, which is not sufficient.
- If it takes the money from shareholders, it must give them \$12,000 each year from its after-tax earnings. The interest payments are tax deductible, so it pays the creditors \$12,000 from its \$15,385 pre-tax earnings, and pays the government $35\% \times (\$15,385 - \$12,000) = \$1,185$ in federal income taxes, leaving it $(1 - 35\%) \times (\$15,385 - \$12,000) = \$2,200$.

We ask: What would be the cost of capital on \$100,000 that leaves \$2,200 in after-tax profit? *Answer:* The payment to the providers of capital is $\$10,000 - \$2,200 = \$7,800$, so the cost of capital must be 7.80%, or $12\% \times (1 - 35\%) = 7.80\%$.

Part B: The income before interest and taxes is \$5 million. Interest is $8\% \times \$10 \text{ million} = \$800,000$, so income before taxes is \$4.2 million.

Question: The textbook speaks of earnings before interest and taxes (EBIT), earnings before interest, taxes, amortization, and depreciation (EBITAD), and earnings before taxes (EBT). Is there some order of payments, as though the firm pays interest on December 28, taxes on December 29, amortization on December 30, and depreciation on December 31?

Answer: The word *before* means *without the deduction for*. Interest is paid throughout the year, not on specific dates. Taxes are usually paid quarterly in advance, and a final adjustment is made in mid-March of the following year. Amortization and depreciation are not cash flows; they are not paid at any time; they are accounting entries. The cash flow is made when the item is bought or the expense is incurred, assuming that payment is made at that time.

- EBIT says how much *earnings* are available to pay the providers of debt capital.
- EBITAD says how much *cash* is available to pay the providers of debt capital.

Question: The textbook says to use the market value of debt and equity, not the book value or par value. Why are we using par value here?

Answer: We use market value to determine the weighted average cost of capital. The earnings before interest and taxes is an accounting item, for which we use book value. In this exercise, the debt is trading at par, so the market value equals the book value.

Question: The textbook says to use the market value to determine the WACC, but many of the examples have the market value the same as the book value. If the objective is to show how to use market value instead of book value, shouldn't the text use more examples where the two values differ?

Answer: Using market values adds complications, particularly if the debt is not perpetual. We want to show the capital structure theory, not to get bogged down in valuation of debt. The text uses examples where the market value is the same as the book value to simplify the math and focus on the financial theory.

This course is similar. The practice problems and illustrative test questions may use complex mathematics. The homework assignments may ask you to explain concepts. The final exam questions avoid the mathematics and focus on the basic ideas. You are tested on your understanding of the text, not on your mathematical ability.

Part C: The marginal tax rate is 35%, so the after-tax income available to shareholders is $\$4.2 \text{ million} \times (1 - 35\%) = \2.73 million .

Question: Don't we add the tax shield?

Answer: The tax shield is $35\% \times 8\% \times \$10 \text{ million} = \$280,000$. If the interest were not tax deductible, the after-tax income to shareholders would be $(1 - 35\%) \times \$5 \text{ million} - \$800,000 = \$2.45 \text{ million}$. Adding the tax shield gives $\$2.45 \text{ million} + \$0.28 \text{ million} = \$2.73 \text{ million}$.

Part D: The market value of equity is $5 \text{ million shares} \times \$5 \text{ per share} = \$25 \text{ million}$.

Part E: The return on equity capital is $\$2.73 \text{ million} / \$25 \text{ million} = 10.92\%$. The return on equity capital is the residual income to shareholders divided by the market value of equity. We compare the return on equity capital to the cost of equity capital to determine if shareholders receive an adequate return.

Part F: The after-tax weighted average cost of capital is $(5.2\% \times \$10 \text{ million} + 10.92\% \times \$25 \text{ million}) / \$35 \text{ million} = 9.29\%$.

We solved the problem by separately evaluating the costs of debt capital and equity capital and we used the weighted average cost of capital. We could also solve the problem directly. The after-tax weighted average

cost of capital is the after-tax income the firm must provide to suppliers of capital, both stockholders and bondholders.

The firm earns \$5 million per annum, and it has \$10 million + \$25 million = \$35 million of capital. Its pre-tax cost of capital is $\$5 \text{ million} / \$35 \text{ million} = 14.29\%$. Its after-tax cost of capital is $14.29\% \times (1 - 35\%) = 9.29\%$.

Question: If the calculation is this simple, why do we use the complex method above?

Answer: Many problems deal with the required returns to shareholders at different debt levels. For these problems, we separately evaluate the returns to debt and equity.

Question: This alternative calculation takes the total return divided by the total capital and multiplies by the complement of the corporate tax rate to get the after-tax return on assets. This module says that the return does not depend on the debt-to-equity ratio if the corporate tax rate is zero, but it does depend on the corporate tax rate if the corporate tax rate is non-zero. Yet this exercise shows the opposite. Suppose the firm trades some of its equity for more debt. The total earnings before interest and taxes of \$5 million stays the same, so the after-tax cost of capital stays 9.29%. What am I missing?

Answer: As the debt-to-equity ratio increases, the value of the firm increases, if the corporate tax rate is positive and there are no other capital market imperfections. With \$10 million of debt, the value of the firm is \$35 million; with more debt, the value of the firm may be \$36 million, and the after-tax return on assets may be $(1 - 35\%) \times \$5 \text{ million} / \$36 \text{ million} = 9.03\%$.

Question: The return on assets decreases when the debt-to-equity ratio increases? Doesn't a lower return on assets mean the assets are worth less, not more?

Answer: This is the theoretical after-tax return assuming everything is taxed. Since the value of the assets increases and the pre-tax income stays the same, the return decreases.

Exercise 18.2: Capital Structure

(Adapted from question 28 of the Spring 1999 examination)

A firm has the following financial attributes:

Liabilities stated at market value are

- Long-term debt = \$100 million
- Short-term debt = \$5.5 million
- Outstanding equity = \$50 million

Opportunity costs (or returns on long-term debt, short term debt, and equity):

- Long-term debt = 10%
- Short-term debt = 7%
- Equity = 15%

The corporate tax rate is 30% and the current ratio is 1.40.

- What is the after-tax cost of long-term debt?
- If we ignore short-term debt, what is the weighted average cost of capital?
- What is the after-tax cost of short-term debt?
- If we include short-term debt, what is the weighted average cost of capital?
- Why might we ignore short term debt in this scenario?

Question: What is the *current ratio*?

Answer: The *current ratio* is *current assets* divided by *current liabilities*. *Current assets* are short-term assets like cash, marketable securities, receivables, and inventory. *Current liabilities* are short term liabilities, like bank loans and accounts payable.

Question: In which module was this?

Answer: This is in module 24; we have not covered it yet.

Solution 18.2:

Part A: The after-tax cost of long-term debt capital is $10\% \times (1 - 30\%) = 7\%$.

Part B: If we ignore short-term debt, then long-term debt is two thirds of capital and equity is one third of capital. The after-tax weighted average cost of capital is

$$\frac{2}{3} \times 10\% \times (1 - 30\%) + \frac{1}{3} \times 15\% = 9.67\%.$$

Part C: The after-tax cost of short-term debt capital is $7\% \times (1 - 30\%) = 4.9\%$.

Part D: If we include short-term debt, the after-tax weighted average cost of capital is

$$100/155.5 \times 7\% + 5.5/155.5 \times 4.9\% + 50/155.5 \times 15\% = 9.50\%.$$

Part E: Since the current ratio is 1.40, the short term debt supports short-term assets. For example, the accounts payable may support accounts receivable, and short term bank loans may support inventory. If the financing for short term debt is similar to the financing for short term assets, the debt offsets the assets. If the short term assets are about the same size as the short term liabilities and they have similar payment terms, we ignore both of them when estimating the costs of other projects. The weighted average cost of capital is the cost of financing the long-term projects of the company, not the short-term assets.

Illustration: If a firm has increased sales of \$5 million one quarter, causing increased accounts receivable of \$3 million, it may need a short-term bank loan to provide \$3 million of short term financing. If the firm decides to build a new plant, requiring \$20 million of construction costs, it would issue long-term debt or new equity.

Question: The textbook is not clear. Should we use all debt or just long-term debt?

Answer: The answer depends on how the firm finances its projects. If it finances projects with short term debt, we include short term debt. If it uses the short term debt to cover its working capital needs and finances projects with long-term debt, we use only long-term debt.

Exercise 18.3: WACC and CAPM

- A firm's market value liabilities are \$40 million of debt and \$80 million of equity.
 - The debt has an expected return of 9% and a β of 0.30; the equity has a β of 1.65.
 - The company has a weighted average cost of capital of 14%, and *the tax rate is zero*.
- A. What is the return on the firm's equity? (Derive this return from the WACC formula. We are given the after-tax return on assets, the pre-tax return on debt, and the tax rate; we back out the return on equity. This problem is simplified by using a 0% tax rate.)
- B. Given the returns on debt and equity and the betas of debt and equity, what are the risk-free interest rate and the market risk premium? (Given the betas and returns of two securities, we solve a pair of simultaneous linear equations for the risk-free rate and the market risk premium.)
- C. Given the risk-free interest rate, the market risk premium, and the return on assets R_A , what is the beta of assets? (R_A is the weighted average cost of capital.)
- D. Verify that your beta of assets is correct by taking the weighted average of the betas of debt and equity.

Solution 18.3:

Part A: The cost of equity capital r_E is determined as

$$\begin{aligned} \frac{1}{3} \times 9\% + \frac{2}{3} \times r_E &= 14\% \Rightarrow \\ r_E &= 1.5 \times (14\% - \frac{1}{3} \times 9\%) = 16.50\% \end{aligned}$$

Since the tax rate is zero, we don't need to adjust the return on debt capital by the tax rate.

Part B: We solve for the risk-free interest rate r_f and the market risk premium MRP as

- Debt: $9.0\% = r_f + 0.30 \times MRP$
- Equity: $16.5\% = r_f + 1.65 \times MRP$

$$\begin{aligned} 7.5\% &= 1.35 \times MRP \Rightarrow MRP = 7.5\% / 1.35 = 5.56\%. \\ r_f &= 9.0\% - 0.30 \times 5.56\% = 7.33\% \quad \ll \text{or} \gg \\ r_f &= 16.5\% - 1.65 \times 5.56\% = 7.33\% \end{aligned}$$

Parts C and D: We solve for the beta of assets in either or two ways:

- The overall β for the company's existing business is $\frac{1}{3} \times 0.30 + \frac{2}{3} \times 1.65 = 1.200$.
- The beta of assets = $(14\% - 7.33\%) / 5.56\% = 1.200$

Exercise 18.4: Returns on Debt and Equity

A company invests \$20,000 in a project, which generates cash flows of \$3,000 at the end of each year for 15 years, with the first cash flow one year after the initial investment. At the project's after-tax weighted average cost of capital, its net present value is \$2,500.

- The company is financed with 40% debt and 60% equity.
 - The company's marginal tax rate is 25%.
 - $r_E = 2 r_D$, where r_E is the cost of equity and r_D is the cost of debt.
- A. If the net present value is \$2,500 and the initial investment is \$20,000, what is the present value of the 15 years of cash flows? (The NPV is the present value of the future cash flows minus the present value of the initial investment. Back out the present value of the 15 years of cash flows.)
- B. Given the present value of the future cash flows of \$3,000 per annum, what is the implicit discount rate (the capitalization rate, or the weighted average cost of capital)? Use a financial calculator to compute the answer. (If we know the capitalization rate, we use the annuity formula to get the present value of the cash flows. We can't solve this problem by pencil and paper, and the final exam does not test your expertise with a financial calculator. But the CAS transition exam may test this, so we include this problem.)
- C. If r_D is the cost of debt capital, $2 \times r_D$ is the cost of equity capital, and $r_D \times (1 - 25\%)$ is the after-tax cost of debt capital. Use the weighted average cost of capital that you derived in Part B and the WACC formula to derive the cost of debt capital (r_D).
- D. What is the cost of equity capital (r_E)? (Twice the cost of debt capital.)

Solution 18.4:

Part A: The present value of the project is $\$20,000 + \$2,500 = \$22,500$.

Part B: The project generates cash flows of \$3,000 at the end of each year for 15 years, for a total present value of \$22,500. We use a financial calculator to derive an implied capitalization rate of 10% per annum. {The final exam for the on-line course does *not* require you to use a financial calculator. The CAS transition exam may require financial calculations. Check the CAS web site for the details of the transition exam.}

Part C: The after-tax cost of debt capital is $r_D \times (1 - 25\%)$. We write

$$\begin{aligned} 40\% \times r_D \times (1 - 25\%) + 60\% \times 2r_D &= 10\% \\ 150\% \times r_D &= 10\% \\ r_D &= 6.667\% \end{aligned}$$

Part D: We solve for $r_E = 2 \times 6.667\% = 13.333\%$

Exercise 18.5: Required Return on Equity

A firm's capital structure is \$40 million debt and \$60 million equity. The interest rate on the debt is 8.0%. The corporate tax rate is 35%. The firm invests \$10 million in a project with a net present value of \$2 million, based on perpetual cash flows of \$1.32 million at the end of each year.

- A. What is the present value of the perpetuity of \$1.32 million each year? (This is the original investment plus the net present value.)
- B. What is the implied return on assets (the weighted average cost of capital)?
- C. What is the after-tax return on debt (the pre-tax return times the complement of the tax rate)?
- D. Using the WACC formula, what is the equity investor's required return?

Solution 18.5:

Part A: The present value of the cash flows is \$10 million + \$2 million = \$12 million.

Part B: The return on assets (the WACC) is \$1.32 million / \$12 million = 11.00%.

Part C: The after-tax cost of debt capital is $8.0\% \times (1 - 35\%) = 5.2\%$.

Part D: If r_E is the cost of equity capital, we infer that

$$\begin{aligned} 8.0\% \times (1 - 35\%) \times \$40 \text{ million} + r_E \times \$60 \text{ million} &= 11\% \times \$100 \text{ million} \\ \Rightarrow r_E &= (11\% \times \$100 \text{ million} - 5.2\% \times \$40 \text{ million}) / \$60 \text{ million} \\ &\Rightarrow r_E = 14.87\%. \end{aligned}$$

Question: Normally, we derive the net present value of the project from the cost of debt capital and the cost of equity capital. This exercise does the reverse; it backs out the cost of equity capital from the cost of debt capital and the net present value of the project. Do we ever do this in practice, or is this just for practice problems?

Answer: The cost of debt capital is often clear. If the debt is new and issued at par, we use the coupon rate as the yield to maturity; otherwise we use the coupon rate on bonds from similar firms that are trading at par. The net present value of a project can sometimes be estimated from market values. Most new projects have a net present value of zero; the present value of an existing project can sometimes be estimated from its sale value (if it is for sale). The cost of equity capital depends on the systematic risk of the project. One way of estimating this cost is to back it out from the NPV (market value) of the project.

Exercise 18.6: Debt-Equity Ratio

A project has annual after-tax cash inflows of \$4 million at the end of every year *forever* and a \$27 million after-tax investment up front. With all equity financing, the net present value of the project is -\$2 million. With a 35% tax rate and a 10% cost of debt financing with the debt remaining fixed for perpetuity, the net present value of the project is zero.

- A. What is the present value of the future cash flows with all equity financing? (This is the original investment plus the net present value.)
- B. What is the weighted average cost of capital with all equity financing? (This is the annual return divided by the present value.)
- C. What is the return on equity with all equity financing? (With all-equity financing, the return on equity is the same as the return on assets.)
- D. What is the increase in the present value of the project if the cost of debt financing is 10%? (This is stated in the exercise; it is the difference between the two net present values.)
- E. What is the value of the tax shields if the cost of debt financing is 10%? (This is the increase in the present value of the project.)
- F. What is the amount of debt that gives this present value of tax shields? (The debt times the tax rate is the present value of the tax shields. We assume that nothing else affects the value of the project.)
- G. Given the amount of debt and the revised value of the project (with a zero NPV), what is the amount of equity?
- H. What is the debt-equity ratio at the 10% cost of debt financing? (This is the ratio of debt to equity.)
- I. What are the pre-tax cash flows? (Divide the after-tax cash flows with all equity financing by the complement of the tax rate.)
- J. What are the cash flows after interest payments but before tax payments with 10% debt financing?
- K. What are the after-tax cash flows with 10% debt financing? (Subtract tax payments.) These are the cash flows available to the investors (equityholders).
- L. What is the return on equity with the 10% cost of debt financing? (Divide the annual after-tax cash flows to investors by the amount of equity.)
- M. What is the weighted average cost of capital with the 10% cost of debt financing?

Solution 18.6:

Part A: The present value of the project is \$27 million - \$2 million = \$25 million. The NPV is -\$2 million, so we subtract \$2 million.

Question: Several problems here differentiate between net present value and present value. In practice, we can't always separate the initial investment cleanly from the subsequent cash flows. What if the initial investment is spread out over several years? For capital intensive projects, with manufacturing aircraft or developing new medications, the initial investment may be spread over a dozen years. Why do we separate the initial investment and other cash flows?

Answer: This is heuristic; in practice, we can not always clearly differentiate the initial investment from the subsequent cash flows. But this is an important distinction.

Illustration: A firm produces aircraft in a competitive market. It needs 10 years to build the factory, design the aircraft, build prototypes, and test the aircraft. It then produces and sells the aircraft for the next 20 years. After 20 years, newer models will take over all sales, but the existing aircraft will be serviced for the next 25 years. Many firms service aircraft, and this firm has no competitive advantage in servicing the aircraft.

We graph the net present value of the project over the 55 years. What is the shape of this graph if the firm's expectations are met? What is the progression of the capitalization rate over the 55 years?

Solution: The net present value starts at zero and climbs to its highest point after 10 years, when the initial investment is completed. It then declines over the next 20 years back to zero, as the firm collects the profit from its investment. The last 25 years of service are in a competitive market, so the net present value is zero.

The capitalization rate depends on systematic risk, so we can't give a definite answer. In practice, the capitalization rate is high at the beginning, when the success of the project is unknown and declines steady for the first 30 years, as the project succeeds. For the last 25 years, the service business is not very risky, so the capitalization rate is low.

Part B: The weighted average cost of capital, or the required return for the project, or the opportunity cost of capital with all equity financing, is:

$$r_A = \$4 \text{ million} / (\$27 \text{ million} - \$2 \text{ million}) = 16\%.$$

Part C: With all equity financing, the return on equity is the return on assets of 16%.

Part D: The increase in the value of the project is \$2 million, since its NPV is now zero.

Part E: The present value of the tax shields is \$2 million.

Part F: The amount of debt is $\$2 \text{ million} / 35\% = \5.71 million .

Part G: The total project is worth \$27 million, so the equity is the project value minus the debt, or $\$27 \text{ million} - \$5.71 = \$21.29 \text{ million}$.

Part H: The debt-equity ratio is $\$5.71 \text{ million} / \$21.29 \text{ million} = 26.82\%$.

Part I: The after-tax cash flows of \$4 million correspond to pre-tax cash flows of $\$4 \text{ million} / (1 - 35\%) = \$6.154 \text{ million} = \text{earnings before interest and taxes}$.

Part J: We subtract the interest payments at a 26.82% debt-equity ratio:

$$\$6.154 \text{ million} - \$5.71 \text{ million} \times 10\% = \$5.583 \text{ million}$$

Part K: The after-tax cash flows to shareholders (the after-tax income to shareholders) is $\$5.583 \text{ million} \times (1 - 35\%) = \3.629 million .

Part L: The return on equity r_E is $\$3.629 \text{ million} / \$21.29 \text{ million} = 17.05\%$.

Part M: The weighted average cost of capital is

$$(\$5.71 \text{ million} \times (1 - 35\%) \times 10\% + \$21.29 \text{ million} \times 17.05\%) / \$27 \text{ million} = 14.82\%.$$

Question: The weights should equal one. In this illustration, $\$5.71 \text{ million} + \$21.29 \text{ million} = \27 million , but $\$5.71 \text{ million} \times (1 - 35\%) + \$21.29 \text{ million} \neq \27 million .

Answer: The $\$5.71 \text{ million} \times 35\%$ is contributed by the taxing authorities (tax exemption). The firm gains by using tax exempt financing.