Corporate Finance, Module 20: Introduction to Options

Put Call Parity Relation

(The attached PDF file has better formatting.)

Upper bonds on call and put options are high. The upper bounds are high because as the volatility of the stock price increases, the value of the option increases. The upper bounds effectively assume that the volatility of the stock price is unlimited. The lower bounds effectively assume that the volatility of the stock price is zero.

CALL OPTIONS: UPPER BOUNDS

Call options are bounded from above by the current value of the underlying security. For example, a call option on a stock currently worth S_0 is worth no more than S_0.

The no-arbitrage demonstration is as follows. Consider two portfolios.

A. Portfolio A consists of one share of stock S.
B. Portfolio B consists of one call option on stock S with strike price X.

At the exercise date, stock S is worth S_T. The value of the call option depends on the relationship between S_T and X. If $S_T > X$, then the call option is worth $S_T – X$, which is less than S_T. If $S_T < X$, then the call option is not exercised and it expires worthless.

Thus, portfolio A is always equal to or greater than portfolio B at the exercise date, so it must be equal to or greater than portfolio B at the current time as well.

We can restate this argument as follows. A call option increases in value as the strike price decreases. That is, if $X_0 < X_1$, then a call on stock S with a strike price of X_0 is worth at least as much as a call on stock S with a strike price of X_1.

If the strike price X to zero, the call option is equal to the value of the stock, since it will be exercised at the exercise date for a strike price of zero. Thus, the stock price is an upper bound, since a call option with a positive strike price is less valuable than a call option with a strike price of zero.

One is tempted to say: *Surely this is not the lowest upper bound. A call option with a strike price of $50 is worth less than a call option with a strike price of $0. Since a call with a strike price of zero is worth S, a call with a positive strike price must be worth less than S.*

In fact, this is indeed the lowest upper bound that we can set. To see this, remember that as the stock's volatility increases, the value of the call option increases. If we let the volatility increase without bound, the value of the call option equals the value of the stock.

One sees this from the Black-Scholes formula. As $\sigma \to \infty$, $d_1 \to \infty$, $N(d_1) \to 1$, $d_2 \to -\infty$, $N(d_2) \to 0$, and the price of the call option $\to S_0$.

This still seems counterintuitive to some candidates. One is tempted to say:

> *When the volatility is infinite, the stock can take any value from zero to infinity at time T. If the stock price is less than the strike price at the exercise date (time T), the call option is worth zero. If the stock price is more than the strike price at the exercise date, the call option is worth the stock price less the strike price. Unless the stock price is zero at the exercise date, the stock is worth more than the call option. So why is the stock not worth more than the call option at the present time?*

Let's provide some numbers to better understand this question. Suppose that the stock price is \$50, and the strike price is \$50. The upper bound is \$50, not anything less.

Compare the option to a forward contract. The forward contract requires the investor to pay the strike price at the exercise date. The value of the forward contract is $S_0 - (e^{-rt})\,X$. The option is a one sided contract. The investor exercises the option only if the payoff at the expiration date is positive. The greater the volatility, the more valuable is the one-sidedness of the contract.

For the intuition, think of the binomial tree pricing method. As the volatility increases to infinity, the up movement become infinite and the down movement goes to zero. The upward movement is $e^{\sigma\sqrt{t}}$, which $\rightarrow \infty$ as $\sigma \rightarrow \infty$, and the downward movement is $e^{-\sigma\sqrt{t}}$, which $\rightarrow 0$ as $\sigma \rightarrow \infty$.

The risk-neutral probability of an upward movement, π, is $(e^{r\sqrt{t}} - e^{-\sigma\sqrt{t}})/(e^{\sigma\sqrt{t}} - e^{-\sigma\sqrt{t}})$. This ratio goes to zero (asymptotically) as $\sigma \rightarrow \infty$.

- The value of the stock is $S_0 = \pi \times$ up value + $(1 - \pi) \times$ down value.
- The value of the call option is $c = \pi \times$ (up value – strike price).

Both the down value and the risk-neutral probability are approaching zero. The difference between the stock price and the call option is $\pi \times$ strike price, which goes to zero as well.

PUT OPTION

For a put option, the upper bound on the price of the option is the present value of the strike price at the risk-free interest rate. The rationale is the same as for the call option; we can replicate each of the arguments given above.

For the no-arbitrage argument, compare two portfolios.

- Portfolio A consists of the present value of the strike price.
- Portfolio B consists of the put option.

Portfolio A is worth at least as much as Portfolio B at the exercise date, so it is worth as least as much now as well.

Similarly, the put option value increases as the current stock price decreases. If the current stock price is \$0, the put option is worth the present value of the strike price. Since the put option might equal the present value of the strike price, this is the lowest upper bounds.

For a second way to see that this is the lowest upper bound that we can determine, examine the Black-Scholes formula. As $\sigma \rightarrow \infty$, $-d_1 \rightarrow -\infty$, $N(-d_1) \rightarrow 0$, $d_2 \rightarrow \infty$, $N(-d_2) \rightarrow 1$, and the price of the put option $\rightarrow Xe^{-rt}$.

The intuition using the binomial tree pricing method is similar to the intuition for call options. The value of π, the risk-neutral probability of an up movement, goes to 0 as the volatility increases to infinity. The value of $(1 - \pi)$ goes to unity, and the value of the stock in the down movement goes to 0. The value of the put option goes to Xe^{-rt}.

LOWER BOUNDS

The lower bound for a call option is the stock price minus the present value of the strike price, if this value is positive; otherwise the lower bound is zero. Symbolically, $c \geq \max(S_0 - Xe^{-rt},0)$.

The no-arbitrage intuition is as follows. Consider two portfolios:

A. Portfolio A consists of one share of the stock.
B. Portfolio B consists of one call option plus cash equal to the present value of the strike price.

At the exercise date, portfolio A equals S_T, the value of the stock. Portfolio B at the exercise date consists of the call option plus cash equal to the strike price (not discounted), since the cash in Portfolio B has accumulated to the strike price.

If the stock price exceeds the strike price, the call option is exercised and the cash equal to the strike price is traded for the stock. If the stock price is less than the strike price, the call option is not exercised, and the value of Portfolio B equals the strike price.

In either case, Portfolio B is worth at least as much as Portfolio A. Thus,

$$\text{call} + Xe^{-rt} \geq S_0, \text{ or } c \geq S_0 - Xe^{-rt}.$$

We show that this is the highest lower bound by noting that if $S_0 > Xe^{-rt}$ and $\sigma \to 0$, the call is a forward contract on stock S. Its value is $(S_T - X) \times e^{-rt} = S_0 - Xe^{-rt}$.

PUT OPTIONS

The lower bound for a put option is the present value of the strike price minus the stock price, if the current stock price is less than the present value of the strike price; otherwise the lower bound is zero. Symbolically, $p \geq \max(Xe^{-rt} - S_0,0)$.

The no-arbitrage intuition is as follows. Consider two portfolios:

A. Portfolio A consists of cash equal to the present value of the strike price.
B. Portfolio B consists of one put option plus one share of the stock.

At the exercise date, we compare the two portfolios. Portfolio A equals X, the value of the strike price (not discounted), since the cash in Portfolio A has accumulated to the strike price. Portfolio B at the exercise date consists of the put option plus the stock.

If the stock price exceeds the strike price, the put option is allowed to expire unexercised. Portfolio B is the stock, which is more than the strike price. If the stock price is less than the strike price, the put option is exercised, and the value of Portfolio B equals the stock price.

In either case, Portfolio B is worth at least as much as Portfolio A. Thus,

$$\text{put} + S_0 \geq Xe^{-rt}, \text{ or } p \geq Xe^{-rt} - S_0.$$

We show that this is the highest lower bound by setting σ equal to zero and comparing the put option with a forward contract, as we did above.

PROBLEMS

Exercise 20.1: What is a lower bound for the price of a four-month call option on a non-dividend-paying stock when the stock price is \$28, the strike price is \$25, and the risk-free interest rate is 8% per annum?

A four month option has $t = \frac{1}{3}$. The formula for the lower bound gives

$$c \geq \$28 - \$25e^{--(0.08)(\frac{1}{3})} = \$3.66$$

Exercise 20.2: What is a lower bound for the price of a six-month call option on a non-dividend-paying stock when the stock price is \$80, the strike price is \$75, and the risk-free interest rate is 10% per annum?

A six month option has $t = \frac{1}{2}$. The formula for the lower bound gives

$$c \geq \$80 - \$75e^{--(0.10)(\frac{1}{2})} = \$8.66.$$

Exercise 20.3: What is a lower bound for the price of a one-month European put option on a non-dividend-paying stock when the stock price is \$12, the strike price is \$15, and the risk-free interest rate is 6% per annum?

A one month option has $t = 1/12$. The formula for the lower bound gives

$$p \geq \$15e^{-0.06/12} - \$12 = \$2.93.$$

Exercise 20.4: What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is \$58, the strike price is \$65, and the risk-free interest rate is 5% per annum?

A two month option has $t = 1/6$. The formula for the lower bound gives

$$p \geq \$65e^{-0.05/6} - \$58 = \$6.46.$$

DIVIDENDS

The intuition for dividends is similar for various topics: forward contracts, put call parity, binomial tree pricing, and the Black-Scholes formula. We deal with two types of dividends.

A. *A dollar dividend paid at a known time in the future.* Most authors treat stock dividends as risk-less cash flows that are known with certainty, so they are discounted at the risk-free interest rate. In truth, there is some uncertainty in the dividend payment, and this uncertainty is related to the price of the stock at the ex-dividend date. The risk-free interest rate is not necessarily the correct discount rate. However, discounting at the risk-free interest rate gives a reasonable approximation.

B. *A percentage dividend that is paid continuously.* Although no individual stock has a dividend that is paid continuously, this is a reasonable perspective for stock indices in the United States, since there are several hundred stocks in the stock index, all with different ex-dividend dates. (This is not an appropriate model for Japanese stock indices, since Japanese stocks pay dividends on the same dates.) This is also an apt perspective for forward contracts, futures contracts, and options on currencies.

Consider first a dollar stock dividend that will be paid on a future date. Two comments on terminology are warranted. First, by "payment date" we mean the ex-dividend date. The ex-dividend date is the date used to determine who gets the stock dividend. The dividend is paid to the owner of the stock on the ex-dividend date, even if that person sold the stock before the dividend was actually paid.

Second, there are potential tax adjustments that are needed, since stock dividends are taxed at a different rate from the rate applied to capital gains. (For personal taxpayers, dividends are taxed at the regular tax rate and capital gains – if they are long-term – are taxed at 20%.) For problems involving dividends, either think of the dividends as the *tax-affected* dividend or ignore income taxes. The *tax-affected* dividend is the amount of the stock price decline after the ex-dividend date.

The stock price declines by the amount of the (tax-affected) dividend on the ex-dividend date. No-arbitrage arguments provide the rationale. Suppose that a stock is worth S_T right before the ex-dividend date and the dividend is DIV. If the expected value of the stock right after the ex-dividend date is greater than S_T – DIV, then an arbitrageur would buy the stock right before the ex-dividend date (paying S_T), receive the dividend of DIV, then sell the stock and receive an amount greater than S_T – DIV. This would provide a positive profit for no work.

Similarly, if the expected value of the stock right after the ex-dividend date is less than S_T – DIV, the arbitrageur would sell short the stock right before the ex-dividend date, pay the dividend from the proceeds to the person from whom he borrowed the stock, then repurchase the stock in the market right after the ex-dividend date. This provides a positive profit for no work.

For European options (as well as for forward contracts and futures contracts), we are concerned with the value of the underlying security at the exercise date, not on any prior date. To grasp the intuition for dividend paying stocks, separate the stock into two pieces: a non-dividing paying stock portion and a dividend portion, and suppose that one can purchase or sell the two parts separately. This is like stripping the coupons from a Treasury security to create a zero coupon bond or like separating the interest payments from the principal repayments of a mortgage to create IO (interest only) strips and PO (principal only) strips.

For example, suppose a stock with a current price of \$100 will pay a \$2 dividend in three months. We think of the stock as two pieces: a non-dividend paying stock worth \$Y and a dividend piece worth \$Z. The dividend piece provides \$2 in three months, so its present value is $\$2e^{-r(0.25)}$. The other piece – the non-dividend paying stock piece – is worth the remainder, or $\$100 - \$2e^{-r(0.25)}$.

The non-dividend paying stock piece is the only part relevant for European options with an exercise date more than 3 months in the future. The rule for valuing such options is simple: adjust the current stock price by removing the present value of future dividends.

The intuition for proportional dividends that are paid continuously is as follows: Suppose a stock index pays a continuous 2% dividend. For European options, we are concerned with the price of the stock at the exercise date. Conceive of the dividends are being reinvested continuously in the stock. A one year European option on one share of the stock index may be viewed as a one year European option on $e^{-2\%}$ shares of the non-dividend portion of this stock, with the dividends used to purchase additional pieces of the share.

LOWER BOUNDS

The formula for lower bonds on the option prices requires that we adjust the current price of the stock by the present value of future dividends. For dollar dividends whose values are known today, we subtract the present value of the dividends. For percentage dividends with a constant dividend rate θ, we multiply the stock price by $e^{-\theta t}$.

ARBITRAGE

The problem will give the price of the stock and the price of either the call option or the put option.

The stock itself cannot be over-valued or under-valued. That is to say, it might be overvalued or undervalued based on an examination of the present value of future dividends. However, that is a subject for investment analysis of primary securities. The presumption of market efficiency can not tell us if the stock itself is overvalued or undervalued. There are no clear arbitrage opportunities based on an examination of future expected dividends.

The options on the stock may be overvalued or undervalued, based solely on arbitrage considerations. The arbitrage opportunity consists of buying undervalued options and selling short overvalued options (that is, writing the overvalued option).

Since the upper bounds on option values are so high, do not expect the exam problem to give prices of overvalued options. Rather, expect to be given prices for undervalued options.

The arbitrage opportunity consists of buying the undervalued option. To complete the arbitrage portfolio, do the following:

A. If you purchase an undervalued call option, you must sell short the stock itself and invest the net proceeds at the risk free rate.
B. If you purchase an undervalued put option, you must also purchase the stock itself, and you must borrow the present value of the exercise price.

PROBLEMS: CALL OPTION ARBITRAGE

> *Exercise §.x:* A four-month European call option on a dividend-paying stock is currently selling for $5. The stock price is $64, the strike price is $60, and a dividend of $0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

We find the lower bound for the call option price, and we check the arbitrage opportunities.

We adjust the stock price by subtracting the present value of future dividends. Since the risk-free interest rate is 12% per annum for all maturities, the present value of the dividend is $\$0.80e^{-(0.12)/12} = \$0.80e^{-(0.01)} = \$0.79$.

If the risk-free interest rate differed by maturity – that is, if the term structure of interest rates were not flat – we would use the risk-free interest rate for a one month maturity.

The price of the equivalent non-dividend paying stock is $64 – $0.79 = $63.21. The lower bound for the four month call option is

$$\$63.21 - \$60e^{-(0.12)/3} = \$63.21 - \$57.65 = \$5.56.$$

This lower bound is greater than the market price of the call option ($5.00), so the call option is undervalued.

> *Since the call option is undervalued, we purchase the call option, short the stock, and invest the net proceeds at the risk-free interest rate.*

Step 1: We short the stock. This means that we borrow the stock from a friend (such as a broker). We agree to repay the stock in four months, and also to pay any interim dividends. We then sell the borrowed stock to a third party for $64.

Step 2: We purchase a call option for its market price of $5.00. This leaves us $59, which we invest at the risk-free interest rate of 12% per annum.

To make the intuition clear, conceive of this as though we make two separate investments. One investment is $0.79 in bank account A, which is used to pay the dividend to the friend from whom we borrowed the stock. The other investment is the remaining $58.21 in bank account B, which accumulates for four months.

Step 3: After one month, the $0.79 has accumulated to $0.80. We use this cash to pay the dividend to the friend from whom we borrowed the stock.

Step 4: After four months, the remaining $58.21 has accumulated to $60.59. We check the price of the stock at this time. If the stock is worth more than $60, we exercise the call option, we purchase the stock for $60, and we return it to the friend from whom we borrowed it. We are left with a net profit of $0.59. If the stock is worth less than $60, we purchase the stock in the market (letting the call option expire unexercised), and we return it to the friend from whom we borrowed it. We are left with a net profit greater than $0.59.

PUT OPTION ARBITRAGE

> Exercise §.x: A one-month European put option on a non-dividend-paying stock is currently selling for \$2½. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur?

The lower bound for the value of the put option is

$$\$50e^{-(0.06)/12} - \$47 = \$47.75 - \$45.00 = \$2.75.$$

This is more than the market price of the put option (\$2.50). For the arbitrage opportunity, we purchase the put option and the stock itself, and we borrow the needed cash at the risk-free interest rate.

Step 1: We purchase the put option for \$2.50 and we purchase the stock itself for \$47. To pay for these purchases, we borrow \$49.50 from a bank at the risk-free interest rate.

Step 2: After one month, the loan will have accumulated to \$49.74. We check the price of the stock. If the stock is worth less than \$50, we exercise the put option and sell the stock for \$50. We use \$49.74 to repay the loan, and we keep \$0.26 as net profit. If the stock is worth more than \$50, we do not exercise the option. Instead we sell the stock in the market, we repay the loan for \$49.74, and we net at least \$0.26 in profit.

Put Call Parity

The lower bounds discussed above are special cases of put call parity. The prices of call options and put options must be non-negative. To determine lower bounds, we set either the call option value or the put option value to zero, and we solve for the lower bound of the other option.

The general relationship between call options and put options that have the same strike price is *call* + *cash* = *put* + *stock*. The *cash* is the present value of the strike price. The call option and the put option must have the same exercise date and the same strike price. The options are European options, not American options. The *stock* is the underlying asset for the call option and the put option; it may be a stock, a stock index, a bond, a commodity, or a foreign currency.

Consider two portfolios:

- Portfolio A consists of a call option plus the present value of the strike price: $c + Xe^{-rt}$
- Portfolio B consists of a put option and one share of the underlying security: $p + S_0$

Consider the values of the two portfolios at the exercise date. The cash in Portfolio A has accumulated to the strike price.

1. If the stock price at the exercise date exceeds the strike price, then we exercise the call option in Portfolio A, using the cash which has accumulated to the strike price. We end up with one share of the stock and no cash. In Portfolio B, the put option expires unexercised, since the stock price is greater than the strike price. We are left with one share of the stock in Portfolio B as well.

2. If the stock price is less than the strike price at the exercise date, the call option in Portfolio A expires unexercised. We are left with the cash, which has accumulated to the strike price. In Portfolio B, we exercise the put option; we give up the one share of stock and we get the strike price in cash.

Portfolios A and B are equal at the exercise date no matter what happens to the stock price. By the law of one price, they must also be equal now. Were they not equal now, arbitragers would purchase the less expensive portfolio and sell short the more expensive portfolio. On the exercise date, they would reverse the transactions and make a sure profit with no investment.

BOUNDS

Earlier in this study aid, we discussed the lower bounds on option prices. These lower bounds may be derived from the put call parity relationship. The call option and the put option must have non-negative values. Using a zero as the value of either the call option or the put option in the put call parity relationship gives the lower bound for the other option.

DIVIDENDS

Dividends are treated in the fashion described earlier. We must adjust the current stock price for anticipated dividends. For dollar dividends, we subtract the present value of the dividends from the current value of the stock. For constant percentage dividends at a continuously compounded rate φ, as occur in problems relating to stock indices and foreign currencies, we adjust the stock price to $S_0e^{-\varphi t}$.

PROBLEMS: PUT CALL PARITY PRICING

> Exercise 20.5: The price of a European call that expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and in five months. The term structure is flat, with all risk-free interest rates being 10%. What is the price of a European put option that expires in six months and has a strike price of \$30? Explain the arbitrage opportunities if the put price is \$3.

This stock has two dividend payments during the term of the option. We adjust the stock price for the present values of both of these dividend payments.

In general, we use the risk-free interest rate appropriate for each cash flow. Certain pricing methods, such as the Black-Scholes formula, assume a constant risk-free interest rate. Other procedures, such as the put call parity relationship, make no assumption about the constancy of the risk-free interest rate.

This problem tells us that the term structure of interest rates is flat. If it were not flat, we would use the two month spot rate to discount the first dividend, the five month spot rate to discount the second dividend, and the six month spot rate to discount the strike price.

We adjust the stock price to $S_0 - \text{PV(DIV)} = \$29 - \$0.50e^{-(0.10)/6} - \$0.50e^{-(0.10)(5/12)}$. The put call parity relationship tells us that

$$\text{put} + \$29 - \$0.50e^{-(0.10)/6} - \$0.50e^{-(0.10)(5/12)} = \$2 + \$30e^{-(0.10)/2}$$
$$\text{or put} = \$2.51.$$

Arbitrage

For the problems regarding lower bounds, we could say that a certain option was undervalued. We bought the option, and we hedged our risk by selling short the replicating portfolio, thereby making a risk-free profit.

For problems with put call parity, we generally don't know whether a given option is under-priced. However, we can say that the put option is underpriced *relative* to the call option, or vice versa.

In this problem, the market price of the put option is \$3 when its implied value from the call option price is \$2.51. The put option is overpriced relative to the call option.

It is possible that the put option is overpriced and the call option is either underpriced or priced correctly. It is possible that the call option is underpriced and the put option is either overpriced or priced correctly. It is possible that both options are overpriced, but the put option is overpriced more (in dollar terms, not in percentage terms). It is possible that both options are underpriced, but the call option is underpriced more.

We don't know which of these scenarios is true. It doesn't matter. We can make a riskless profit by purchasing the call option and simultaneously selling (writing) the put option.

When faced with these problems on the exam, begin with the put call parity relationship:

$$\text{call} + \text{cash} = \text{put} + \text{stock}.$$

We purchase the call and sell short (or write) the put. We purchase or sell the entire portfolios, not just the options. That is, we purchase "call + cash" and we sell short "put + stock." To purchase "cash" means to purchase a risk-free bond, or to invest cash in the risk-free bond market. To sell "cash" short means to borrow money at the risk-free interest rate.

The arbitrager proceeds as follows:

Step 1: We sell short the stock. That is, we borrow the stock from a friend (such as a broker) and sell it to a third party for $29.

Step 2: We sell a put option. That is, we write a put option. This gives us an additional $3, for a total of $32.

Step 4: We purchase the call option for $2, leaving us $30. We invest this remaining cash for two months at the risk-free interest rate of 10% per annum, giving us $\$30e^{0.10/6} = \$30.50$.

Step 5: We pay the first $0.50 dividend to the friend from whom we borrowed the stock. We invest the remaining $30 for three months at the risk-free interest rate of 10% per annum, giving us $\$30e^{0.10/4} = \$30.76$.

Step 6: We pay the second $0.50 dividend to the friend from whom we borrowed the stock. We invest the remaining $30.26 for one month at the risk-free interest rate of 10% per annum, giving us $\$30.26e^{0.10/12} = \$30.51$.

Step 7. It is now the exercise date for both the call option and the put option. We check the price of the stock in the market. If the stock price is greater than $30 (the strike price of the options), we exercise the call option to purchase the stock for $30. We return the stock to the friend from whom we borrowed the stock, and we are left with $0.51 in profit.

If the stock price is less than $30, we let the call option expire unexercised. However, the party to whom we sold the put option will exercise it. We pay $30 and we get one share of the stock. We return this stock to the friend from whom we borrowed the stock. We are left with $0.51 in profit.

Note: In the “lower bound” problems, the final (arbitrage) profit depends on the final value of the stock. In the put call parity relationship problems, the final (arbitrage) profit is the same no matter what the final price of the stock. In this problem, the final profit is exactly $0.51, no matter what the price of the stock.

Exercise 20.6: An investor buys a call with strike price X and writes a put with the same strike price. Describe the investor's position.

PAYOFF

The payoff at the exercise date is a combination of a long call and a short put.

- If $S_T > X$ at the exercise date, we exercise the call option and the payoff is $S_T - X$.
- If $S_T < X$ at the exercise date, the other party exercises the put option. The payoff to us is $-(X - S_T)$, or $S_T - X$.

Thus, in both cases, the payoff to us is $S_T - X$.

To this payoff, we must add or subtract the premium for the options. If the price of the call option is "c" and the price of the put option is "p," we pay "c" and we receive "p." The net premium that we make is "c – p."

The put call parity relationship tells us the value of "c – p." We know that

$$\textit{call} + \textit{cash} = \textit{put} + \textit{stock}, \textit{ or}$$
$$c - p = \textit{stock} - \textit{cash} = S_0 - Xe^{-rt}.$$

In other words, we pay $S_0 - Xe^{-rt}$ when we purchase and write these options, and we receive $S_T - X$ on the exercise date. Both of these amounts may be positive or negative.

What is the present value of the *expected* payoff $E(S_T - X)$? To get the expected value, we must discount the two pieces by the capitalization rate appropriate for each. The present value of $E(S_T)$ is S_0. The present value of X is Xe^{-rt}. Thus, the present value of $E(S_T - X)$ is $S_0 - Xe^{-rt}$.

The value $S_0 - Xe^{-rt}$ represents the present value of a forward contract with a forward price of X. This value may be either positive or negative, depending on the values of X, r, and t.

GRAPHICS

The graph of a long call payoff is a horizontal line along the X-axis until the strike price, then a diagonal line upwards. The graph of a short put payoff is a diagonal line moving up to the X-axis and intersecting at the strike price, then a horizontal line along the X-axis. The graph of a long call plus a short put is 45° diagonal line intersecting the X-axis at the strike price.

To get the graph of the net profit, we must include the net premium. The net premium is $S_0 - Xe^{-rt}$. The effect of including the net premium is to move the intersection point with the X-axis either right or left.

We must be careful when adding the net premium to the net payoff, since they occur at different times. The rest of this paragraph helps you visualize the graph, though it is not quite correct. Suppose first that the risk-free interest rate is zero. The net premium is $S_0 - X$. We move the intersection point with the X-axis from X to S_0. The net profit is simply the graph of a long stock purchase.

This is not quite correct. The payoff has S_T along the horizontal axis. The net premium has S_0. For the accurate interpretation, we must either accumulate S_0 to $E(S_T)$ or we must discount $E(S_T)$ to S_0.