## Homework Assignment

(The attached PDF file has better formatting.)

## Put Call Parity Relation

A European call option that expires in three months and has a strike price of $\$ 80$ has a price of $\$ 5$. The underlying stock price is $\$ 77$, the stock pays no dividends, and the risk-free interest rate is $8 \%$ per annum compounded quarterly, or $2 \%$ each three months.
A. What is the price of a European put option on this stock that expires in three months and has a strike price of $\$ 80$ ? Use the put call parity relation, that $c+P V(X)=p+S$.
B. If the European put option also had a price of $\$ 5$, which would you prefer to buy, the call option or the put option? (A consumer prefers the good that is underpriced, so the true value is more than the price paid.)
C. If the European put option also had a price of $\$ 5$, how might an investor make a risk-free profit? Assume the investor is a large financial intermediary that can borrow at the risk-free rate. The transactions at time 0 are:

- The investor borrows $\$ 77$ from a bank at the risk-free interest rate of $8 \%$ per annum, or $2 \%$ for three months.
- The investors uses the $\$ 77$ to buy one share of stock.
- The investor sells a call option for $\$ 5$.
- The investor buys a put option for $\$ 5$.

1. What is the investor's cash outflow at time 0 ? (The bank loan pays for the share of stock; the proceeds from selling the call option pay for the put option.)
2. In three months time, how much does the investor repay the bank? ( $\$ 77 \times 1.02$ )
3. If the stock price in three months time is less than $\$ 80$, which option is exercised, the put or the call? (In any scenario, either the put option or the call option is exercised, not both.) What does the investor do with his share of stock, and how much cash does he receive? What is the investor's net gain? (The amount received for the share of stock minus the repayment to the bank.)
4. If the stock price in three months time is more than $\$ 80$, which option is exercised, the put or the call? What does the investor do with his share of stock, and how much cash does he receive? (The investor wrote the call option; the buyer exercises it against the investor.) What is the investor's net gain?

Question: If this problem gave the risk-free interest rate as $8 \%$ with (i) annual compounding, (ii) semi-annual compounding, or (iii) continuous compounding, how do we solve it?

Answer: We solve for the interest rate over three months:
(i) Three month rate $=1.08^{1 / 4}-1=1.94 \%$
(ii) The six month rate is $8 \% / 2=4 \%$, so the three month rate $=1.04^{1 / 2}-1=1.98 \%$
(iii) The three month rate with continuous compounding is $8 \% / 4=2 \%$, so the effective three month rate is $\ln (1.02)=1.98 \%$

Question: The net profit is small; is it worth using this investment strategy?
Answer: The investor does this for 100,000 shares, 100,000 put options, and 100,000 call options. A profit of $\$ 1$ per share become $\$ 100,000$ for 100,000 shares.

Question: Don't we have to consider expenses as well?

## Answer: We have two answers:

- The expenses eliminate some of the profit, but not all.
- The investor is an options trader, for whom we are determining marketing strategy. The expenses are paid by the clients of the options trader.

We do not consider expenses in the corporate finance on-line course, though they are discussed in practical investment courses.

