

Corporate Finance, Module 21: "Option Valuation"

Homework Assignment

(The attached PDF file has better formatting.)

Binomial Tree Pricing Method

A firm's current share price is \$80; one year from now, its share price will either fall to \$76 or rise to \$92. The risk-free rate is 5%, and one-year European call and put options on the stock have an exercise price of \$85.

- A. What is the upward movement if the stock price rises to \$92? (Express this as a factor of $\$92 / \80 .) Call this upward movement by the symbol $\underline{U}$.
- B. What is the value of the *call* option at its maturity if the stock price rises to \$92? (The call option gives the investor the right to *buy* the stock for the strike price of \$85. How much would the investor pay to buy a stock for \$85 if its price is \$92?) Call this price Call^+ , or the value of the call option at maturity if the stock price rises.
- C. What is the value of the *put* option if the stock price rises to \$92? (The put option gives the investor the right to *sell* the stock for the strike price of \$85. How much would the investor pay to sell a stock for \$85 if its price is \$92?) Call this price Put^+ , or the value of the put option at maturity if the stock price rises.
- D. What is the downward movement if the stock price falls to \$76? Call this downward movement by the symbol $\underline{D}$.
- E. What is the value of the call option if the stock price falls to \$76? (Call^-)
- F. What is the value of the put option if the stock price falls to \$76? (Put^-)
- G. The *actual probability* of a rise in the stock price is not relevant for options pricing. We discuss it here to differentiate it from the risk-neutral probability. Suppose the expected return on the stock is 12% per annum. The stock has only two possible values at the end of the year, \$92 or \$76. If the probability of rising to \$92 is P , it must be that $P \times U + (1 - P) \times D = 1.120$. We solved for U and D earlier; now solve for P .
- H. What is the *risk-neutral probability* of a rise in the stock price? If all investors are risk-neutral, the expected return from the stock is 5% per annum, not 12% per annum. To determine the risk-neutral probability, solve $P \times U + (1 - P) \times D = 1.050$. We use this value of P in the remaining parts of this homework assignment.
- I. We used U and D as factors; we can also express them as percentage returns. If $U = \$92 / \$80 - 1$ and $D = \$76 / \$80 - 1$, then $P \times U + (1 - P) \times D = 5\%$. (There is nothing to write for this part; it is informative.)
- J. What is the expected value of the call option at its maturity in a risk-neutral world? (We solved for the values of the call option at its maturity if the stock price moves up or down. Using the value of P , solve for the expected value of the call option at its maturity in the risk-neutral world: $P \times \text{Call}^+ + (1 - P) \times \text{Call}^-$)
- K. What is the present value of the call option? (In a risk-neutral world, all discounting is done at the risk-free interest rate. Discount the value obtained above at a 5% rate.)
- L. What is the expected value of the put option at expiration in a risk-neutral world?
- M. What is the present value of the put option?
- N. Verify that the put call parity relation holds. Using the present values of the call option and put option, show that $\text{call} + \text{PV}(\text{exercise price}) = \text{put} + \text{stock price}$.