

Corporate Finance, Module 23: "Advanced Option Valuation"

American Options Illustrative Test Questions and Practice Problems

(The attached PDF file has better formatting.)

{This posting contains more information than is needed for the corporate finance on-line course.}

Exercise 23.1: American Call Option

A two-year American call option on a stock that pays an annual dividend equal to 10% of the share price (with the next dividend payable one year from today) has an exercise price equal to the current share price. Each year the stock may increase in value by 20% or decrease in value by 20%. The current risk-free rate is 5%.

We examine when an investor would exercise the option *prior* to expiration:

- A. If the stock declines to 80% of its value at year 1, why would it not be valuable to exercise the option early?
- B. If the stock price rises in the first year to 120% of its starting value, why would it be valuable for the investor to exercise the option early?

Solution 23.1:

The risk-neutral probability of a rise in the stock price is

$$p = (1.05 - 0.80) / (1.20 - 0.80) = 62.50\%$$

The stock price drops by the dividend when it is paid. The dividend is 10% of the stock price, so the post-dividend stock price is 90% of the pre-dividend stock price. If the stock declines to 80% of its value at year 1, its post-dividend value is $80\% \times 90\% = 72\%$ of its starting value. Even if the stock increases by 20% in the second year, it remains at $72\% \times 120\% = 86.40\%$ of its starting value. This is below the exercise price, and the call option would not be exercised.

If the stock price rises in the first year to 120% of its starting value, the call value right before payment of the dividend is 20% of the starting stock price. If the investor waits, the stock price drops to $120\% \times 90\% = 108\%$ of its starting value when the dividend is paid. The risk-neutral probability that the stock price will rise in the next period is 62.5%. If the stock price rises in the next period to $108\% \times 120\% = 129.60\%$ of its starting value, the call option is exercised; if the stock price falls to $108\% \times 80\% = 86.40\%$ of its starting value, the call option is not exercised.

The value of the call option once it rises during the first year if the investor waits to exercise until maturity is $62.5\% \times 29.6\% / 1.05 = 17.62\%$ of its starting value. It is preferable to exercise the option right before the dividend is paid (at year 1) if the stock price rises and get 20% of the starting stock value.

Question 23.2: American Options

The Black-Scholes formula can be used to *exactly* value which of the following?

- A. An American call option on a non-dividend paying stock
- B. An American call option on a dividend paying stock
- C. An American put option on a non-dividend paying stock
- D. An American put option on a dividend paying stock
- E. None of A, B, C, or D is true.

Answer 23.2: A

The Black-Scholes equation can be used to value European options, not American options. An American call option on a non-dividend paying stock is equal to a European call option on that stock. The other American options in this question may be worth more than the corresponding European options.

For the final exam on the corporate finance course, know that the value of an American call option on a non-dividend paying stock is the same as that of a European call option. This is not true for put options or for call options on dividend paying stocks.

Exercise 23.3: Early Exercise

For which of the following options might it be rational to exercise before maturing?

1. American put on a dividend paying stock
2. American put on a non-dividend paying stock
3. American call on a dividend paying stock

Solution 23.3: 1, 2, and 3

It may always be rational to exercise an American put option before the exercise date, whether the stock is dividend paying or not. To see this, suppose the stock price drops to zero. Since the stock price is the present value of future expected cash flows, a zero stock price means that investors do not expect the stock price to rise above zero in the future. It is optimal to exercise the American put option immediately, since there is no volatility value to holding the option so one might as well get the cash (the strike price) now and invest it.

An American call option should be exercised only right before a dividend payment (if at all). The stock price drops when the dividend is paid, so if the dividend is large enough (and the stock price will drop materially on the ex-dividend date), a rational investor may exercise the option.

In sum, it may be rational to exercise all three of the options listed above before the exercise date. The only option that is never optimal to exercise early is an American call option.

Exercise 23.4: Option Rules

Which of the following statements about options are true?

1. Put/call parity implies that puts and calls should trade at the same price, when the stock price equals the exercise price.
2. Early exercise of an American call option makes sense only when there is positive cash flow prior to maturity on the underlying asset.
3. As price volatility increases, call option prices rise, and put option prices fall.

Solution 23.4: 2 only

The put call parity relation is

$$\text{call option} + \text{present value of exercise price} = \text{stock} + \text{put option}.$$

Statement 1 should say: put/call parity implies that puts and calls should trade at the same price when the stock price equals *the present value* of the exercise price.

Statement 2 is correct: positive *cash flow prior to maturity* means a stockholder dividend.

Statement 3 is false; as price volatility rises, both call and put prices rise.