Corporate Finance, Module 23: "Advanced Option Valuation"

Black-Scholes Practice Problems

(The attached PDF file has better formatting.)

{This posting contains more information than is needed for the corporate finance on-line course.}

Exercise 23.1: Black-Scholes Pricing

A stocks price variance rate, or σ^2, is 25%. (Brealey and Myers sometimes express this as the annual variance of a company's continuously compounded stock price.) The nominal risk-free rate payable quarterly is currently 8%. (8% per annum with quarterly compounding is 2% each quarter.) The company's stock now trades at \$100. Three-month European calls and puts are trading with a strike price of \$108.

A. What are the values of the five input parameters to the Black-Scholes model?
B. What is the value of *ln*(S/PV(X)): the logarithm of the ratio of the current stock price to the present value of the exercise price?
C. What are the values of d_1 and d_2?
D. What are the values of $N(d_1)$, $N(–d_1)$, $N(d_2)$, and $N(–d_2)$?
E. What is the value of the European call option?
F. What is the value of the European put option?
G. Verify that the put call parity relation holds.

Solution 23.1:

Part A: We determine the Black-Scholes parameters:

- σ^2 = 25% per year, so σ = 50%.
- t = 0.25
- r = 8% payable quarterly or 2% per quarter.
- S = \$100
- X = \$108 and PV(X) = \$108 / 1.02 = \$105.88

Part B: *ln*(S/PV(X)) = ln(\$100/\$105.88) = ln(0.944) = -0.057

Part C: The values of d_1 and d_2 are

$$d_1 = \frac{\ln(S / PV(X)) + (\sigma^2 / 2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S / PV(X)) - (\sigma^2 / 2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

d_1 = (-0.057 + ½ × 0.25 × 0.25] / (0.5 × 0.5) = -0.104
d_2 = –0.104 – 0.5 × 0.5 = -0.354

Part D: $N(d_1)$ = N(–0.104) = 0.459; $N(–d_1)$ = N(0.104) = 0.541
$N(d_2)$ = N(–0.354) = 0.362; $N(–d_2)$ = N(0.354) = 0.638

Part E: The value of the call option is \$100 × 0.459 – \$105.88 × 0.362 = \$7.57

Part F: The value of the put option is –\$100 × 0.541 + \$105.88 × 0.638 = \$13.45

Part G: \$7.57 + \$105.88 = \$13.45 + \$100 = \$113.45

Exercise 23.2: Black-Scholes Pricing

- The standard deviation of the continuously compounded annual rate of return on the stock is 0.4.
- The stock price is now $100 and pays no dividends.
- The time to maturity of the option is 3 months (0.25 years).
- *ln* (current share price / present value of the exercise price) = –0.08, at the risk-free rate.

A. What is the present value of the exercise price? (Derive this value from *ln*(S / PV(X)) = –0.08.) This is the one Black-Scholes parameter that we are not explicitly told.
B. What are the values of d_1 and d_2?
C. What are the values of $N(d_1)$, $N(–d_1)$, $N(d_2)$, and $N(–d_2)$?
D. What is the value of the European call option?
E. What is the value of the European put option?
F. Verify that the put call parity relation holds.

Solution 23.2:

Part A: We determine the values of the Black-Scholes parameters:

- S = $100
- t = 0.25
- σ = 0.4
- *ln*(S / PV(X)) = –0.08 ⇛ PV(X) = S / $e^{–0.08}$ = S × $e^{0.08}$ = $108.33

Part B: The values of d_1 and d_2 are

$$d_1 = \frac{\ln(S / PV(X)) + (\sigma^2 / 2)t}{\sigma\sqrt{t}}$$
$$d_2 = \frac{\ln(S / PV(X)) - (\sigma^2 / 2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

d_1 = (–0.08 + ½ × 0.16 × 0.25] / (0.4 × 0.5) = -0.300
d_2 = –0.300 – 0.4 × 0.5 = -0.500

Part C: The values are

$N(d_1)$ = N(–0.300) = 0.382; $N(–d_1)$ = N(0.300) = 0.618
$N(d_2)$ = N(–0.500) = 0.309; $N(–d_2)$ = N(0.500) = 0.691

Part D: The value of the call option is $100 × 0.382 – $108.33 × 0.309 = $4.73

Part E: The value of the put option is –$100 × 0.618 + $108.33 × 0.691 = $13.06

Part F: $4.73 + $108.33 = $113.06 = $13.06 + $100