Corporate Finance, Final Exam, Practice Problems, Beta of Assets and Equity
(The attached PDF file has better formatting.)
*Question 1.1: Beta of Assets and Equity
Suppose the corporate tax rate is $35 \%$ but there are no costs of bankruptcy or other capital market imperfections. The risk-free rate is $8 \%$ per annum and the market risk premium is $4 \%$. The opportunity cost of capital for an all-equity financed firm is $12 \%$ per annum. If the firm issues debt at $8 \%$ per annum and repurchases some shares, which of the following is true?
A. The beta of both assets and equity increase above 1.
B. The beta of both assets and equity decrease below 1 .
C. The beta of assets decreases below 1 and the beta of equity increases above 1 .
D. The beta of assets increases above 1 and the beta of equity decreases below 1 .
E. The beta of assets stays the same and the beta of equity decreases below 1.

Answer 1.1: C
We assume the CAPM holds, so the expected return $=$ the risk-free rate $+\beta \times$ the market risk premium. This equation holds for equity, debt, and assets. The beta moves in the same direction as the expected return. If the expected return increases, the beta increases, and if the expected return decreases, the beta decreases.

- In perfect capital markets, when the corporate tax rate is zero, the Modigliani and Miller Proposition 1 says that the value of the firm and its expected return do not change.
- If capital markets are not perfect (the corporate tax rate is more than zero), the firm receives the present value of the debt tax shields, and its expected return changes.

If the firm takes on debt, its value increases by the present value of the tax shields. Its after-tax income increases by the tax shield in that year. The tax shield divided by the present value of future tax shields is the return on debt capital, which is less than the return on assets. The new return on assets is lower than the old return on assets, so the beta of assets decreases.

Illustration: A firm has $\$ 200$ million of assets. The return on equity capital for an all equity financed firm is $20 \%$, and the risk-free rate is $10 \%$. The all equity financed firm makes $\$ 40$ million per annum after-tax.

If the firm takes on $\$ 10$ million in debt at $10 \%$ and repays $\$ 10$ million of equity, the value of the firm's assets increases to $\$ 203.5$ million. The debt payments are $\$ 1$ million per annum, so the after-tax income increases to $\$ 40.35$ million. The new return on assets is $\$ 40.35$ million $/ \$ 203.5$ million $=19.83 \%$.
*Question 1.2: Beta of Assets and Equity
On April 13, the risk-free rate is $7 \%$, the expected on the overall stock market is $15 \%$, and the expected return on stock ABC is $17 \%$. On April 14, after a meeting of the Federal Reserve Board, the risk-free rate changes to $7.5 \%$. Which of the following is true on April 14? The average risk of stocks has not changed. The risk-free rate has a risk of zero, which also has not changed.
A. The market risk premium changes to $15 \%-7.5 \%=7.5 \%$; the average stock beta changes to $7.5 \% / 8.0 \%$ $=0.9375$.
B. The market risk premium changes to $15 \%-7.5 \%=7.5 \%$; the average stock beta stays 1 .
C. The market risk premium stays $8 \%$; the average stock beta changes to $7.5 \% / 8.0 \%=0.9375$.
D. The market risk premium stays $8 \%$; the average stock beta stays 1 .
E. The market risk premium changes to $15 \%-7.5 \%=7.5 \%$ and the average stock beta changes to $8.0 \%$ $/ 7.5 \%=1.0667$.

Answer 1.2: D
The market risk premium reflects the risk of common stock. It is the additional return required by investors to assume the uncertainty of common stock returns. A change in the risk-free rate does not affect this additional required return, so the market risk premium does not change when the risk-free rate changes.

The market risk premium is an expectation, not a realized value. Its realized value is greatly affected by random fluctuations, it is difficult to measure. Financial economists can not say with certainty that it has been constant for the past hundred years, though we often make that assumption. If the market risk premium is constant, we can take a one hundred year average; this is a common procedure.

We assume that the systematic risk of common stocks might depend on political and economic events, such as wars and depressions, but it should not depend on the actions of the Federal Reserve Board. If investors require an $8 \%$ premium to invest in common stocks instead of risk-free bonds when the risk-free rate is $6 \%$, they require the same premium when the risk-free rate is $6.5 \%$ or $5.5 \%$.

Question: If the risk-free rate rises to $6.5 \%$ and nothing else changes, the firm's cash flows do not change. If the firm earned $14 \%$ before, it should earn $14 \%$ now. Shouldn't the market risk premium decrease to offset the rise in the risk-free rate?

Answer: We consider two scenarios, each of which is reasonable.

- Scenario \#1: If risk-free rates increase 50 basis points, all investors increase their required returns by 50 basis points. The firm increases prices by 50 basis points, and its cash flows move with interest rates.
- Scenario \#2: Risk-free rates increase 50 basis points, but other cash flows do not change. Investors in common stock increase their required returns by 50 basis points. The firm cash inflows do not change, so the firm's present value and stock price decrease.

Question: To measure the relation of the market risk premium to the risk-free interest rate, can we regress the excess of the stock return over the risk-free rate on the risk-free rate?

Answer: In practice, this is not easy. We don't have an accepted measure of the market risk premium. Some financial economists use the return on the stock market; others argue that the return should be on all risky assets, not just stocks. Financial economists argue about the proper risk-free rate: 90 day Treasury bills, LIBOR rates, on long-term bond rates. We assume that the market risk premium moves with economic and political conditions, which also affect the risk-free rate. It is hard to get an accepted regression.

W08: 7cd: Question 1.3: Risk-free Rates, Market Risk Premium, and Beta

On September 20, the risk-free rate is $8 \%$, the return on the overall market is twice the risk-free rate, and oil stocks have an average CAPM beta of one. On September 21, war unexpectedly breaks out in the Middle East and oil supplies are disrupted.

- The risk-free rate increases by 2 percentage points.
- The market risk premium increases by 2 percentage points.
- The expected return on oil stocks increases by 5 percentage points.

What is the new CAPM beta of oil stocks?
A. 1.100
B. 1.125
C. 1.150
D. 1.175
E. 1.200

## Answer 1.3: A

Solution: 1.100

## W08: Question 1.4: Beta of Assets and Equity

The corporate tax rate is $35 \%$ but there are no costs of bankruptcy or other capital market imperfections. The risk-free rate is $8 \%$ per annum and the market risk premium is $4 \%$.

- Firm ABC produces sports gear: snowmobiles, bicycles, treadmills, skis, and weights.
- Firms in this industry with no debt have expected returns of $12 \%$ per annum.
- Firm ABC has a debt-to-equity ratio of $60 \%$.
- The yield on ABC's debt is $9.5 \%$.

Which of the following is true for $A B C ?(<1$ mean less than $1 ;>1$ means more than 1$)$

|  | Beta of Assets | Beta of Equity |
| :---: | :---: | :---: |
| A | $<1$ | $<1$ |
| B | $>1$ | $>1$ |
| C | $<1$ | $>1$ |
| D | $>1$ | $<1$ |
| E | 1 | 1 |

Answer 1.4: C
We assume the CAPM holds, so the expected return $=$ the risk-free rate $+\beta \times$ the market risk premium. This equation holds for equity, debt, and assets. The beta moves in the same direction as the expected return. If the expected return increases, the beta increases, and if the expected return decreases, the beta decreases.

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return on debt capital, which is less than the return on assets. The new return on assets is lower than the old return on assets, so the beta of assets decreases.

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W08: Question 1.5: Beta of Assets and Equity
The corporate tax rate is $35 \%$ but there are no costs of bankruptcy or other capital market imperfections. The risk-free rate is $8 \%$ per annum and the market risk premium is $4 \%$.

- Firm ABC produces sports gear: snowmobiles, bicycles, treadmills, skis, and weights.
- Firms in this industry with no debt have expected returns of $12 \%$ per annum.
- Firm ABC has a debt-to-equity ratio of $60 \%$.
- The yield on ABC's debt is 9.5\%.
- Assume the debt is perpetual.
- On July 1, 20X9, ABC uses retained earnings to retire $\$ 20$ million of debt.
$\square$ ABC's stock price on June 30, 20X9, is $\$ 80$ a share on 1,000,000 shares.
What is ABC's stock price on July 2, 20X9?
(To solve this problem: In perfect capital markets, payoff off debt does not affect the stock price. In this scenario, ABC loses the present value of the tax shields. Work out the present value of the tax shields on $\$ 20$ million of debt and divide by the number of share.)
A. $\$ 60$
B. $\$ 61$
C. $\$ 73$
D. $\$ 78.3$
E. $\$ 80$

Answer 1.5: C

W08: 7cd: Question 1.6: Tax Shields
A firm has perpetual debt at a fixed coupon rate, and it does not intend to vary the amount of the debt. If the corporate tax rate is $35 \%$ and the present value of the tax shields from the debt is $\$ 70,000$, what is market value of the perpetual debt?
A. $\$ 70,000$
B. $\$ 70,000 /(1-35 \%)=\$ 107,692$
C. $\$ 70,000 / 35 \%=\$ 200,000$
D. $\$ 70,000 \times 35 \%=\$ 24,500$
E. $\$ 70,000 \times(1-35 \%)=\$ 45,500$

Answer 1.6: C
With perpetual debt that does not vary, the present value of the tax shields is the corporate tax rate times the market value of the debt.

