Corporate Finance, Final Exam, Practice Problems, Debt-to-Equity Ratio

(The attached PDF file has better formatting.)

*Question 1.1: Debt-to-Equity Ratio

A firm has a 25% debt-to-equity ratio, so one fifth of its long-term capital is debt and four fifths is equity. The yield to maturity on the debt is 7%, and the return expected by shareholders on their equity is 15%. The corporate tax rate is 35%, and there is no difference in personal tax rates between interest income and capital gains. Interest payments on debt are tax exempt; dividends to shareholders are paid from after-tax funds.

What is the weighted average cost of capital (WACC) for this firm?

A.   20% × 7% × (1 − 35%) + 80% × 15% × (1 − 35%) = 8.71%
B.   20% × 7% × 35% + 80% × 15% = 12.49%
C.   20% × 7% × (1 − 35%) + 80% × 15% × 35% = 5.11%
D.   20% × 7% + 80% × 15% × (1 − 35%) = 9.20%
E.   20% × 7% × (1 − 35%) + 80% × 15% = 12.91%

Answer 1.1: E

The weighted average cost of capital (WACC) is the after-tax income needed to pay for a dollar of capital.

~   Shareholders are paid from after-tax income. If they require an R% return, the firm must earn $R% (after-tax) for each dollar of capital.
~   Creditors are paid from pre-tax income. If they require an R% return, the firm must earn $R% (pre-tax) for each dollar of capital, or $R% × (1 − tax rate) in after-tax funds.

The weighted average cost of capital WACC = $R_a = \alpha \times R_d \times (1 − \tau) + (1 − \alpha) \times R_e$,

where α is the percentage of debt, τ is the corporate tax rate, R_a is the return on debt, R_d is the return on debt, and R_e is the return on equity.

*Question 1.2: Debt-to-Equity Ratio

A firm has a 25% debt-to-equity ratio, so *one fifth* (20%) of its long-term capital is debt and *four fifths* (80%) is equity. The yield to maturity on the debt is 7%, and the weighted average cost of capital for the firm is 15%. The corporate tax rate is 35%, and there is no difference in personal tax rates between interest income and capital gains.

What is the return expected by shareholders on the equity?

A.   (15% − 20% × 35% × 7%) / 80% = 18.14%
B.   (15% − 20% × 65% × 7%) / 80% = 17.61%
C.   (15% − 80% × 35% × 7%) / 20% = 56.80%
D.   (15% − 80% × 65% × 7%) / 20% = 65.20%
E.   (35% × 20% − 80% × 7%) / 65% = 2.15%

Answer 1.2: B

The weighted average cost of capital is $R_a = \alpha \times R_d \times (1 − \tau) + (1 − \alpha) \times R_e$, where α is the percentage of debt, R_a is the return on debt, R_d is the return on debt, and R_e is the return on equity. This implies that the return on equity is $R_e = [R_a − \alpha \times R_d \times (1 − \tau)] / (1 − \alpha)$

For this scenario: (15% – 20% × 65% × 7%) / 80% = 17.61%

We verify the solution with the formula for the weighted average cost of capital:

20% × 7% × (1 – 35%) + 80% × 17.61% = 15.00%

*Question 1.3: Debt-to-Equity Ratio

A major cost of bankruptcy is the expected loss in asset value upon dissolution of the firm. Assets that can be sold without losing value have little cost of bankruptcy.

● The assets of pharmaceutical firms are past research and development, patents for new medications, and the scientific knowledge of their staffs. These assets lose value in bankruptcy, so the cost of bankruptcy is high.
● The assets of hotels are buildings in major cities and resorts. These assets do not lose value in bankruptcy, so the cost of bankruptcy is low.

Suppose that hotels and pharmaceutical firms have the same expected return on assets at their optimal debt-to-equity ratios and both types of firms have higher returns on equity than returns on debt. Assume that the optimal capital structure depends on the cost of bankruptcy, and both hotels and pharmaceutical firms are at their optimal capital structures. We infer that

A. Hotels have higher debt-to-equity ratios and higher costs of equity capital.
B. Hotels have higher debt-to-equity ratios and lower costs of equity capital.
C. Hotels have lower debt-to-equity ratios and higher costs of equity capital.
D. Hotels have lower debt-to-equity ratios and lower costs of equity capital.
E. Hotels have lower debt-to-equity ratios and the same costs of equity capital.

Answer 1.3: A

Hotels and pharmaceutical firms opposites: hotels have very low cost of bankruptcy, since

● Their fixed assets are in desirable locations (large cities and resorts) and can generally be sold for their full values.
● Their employees are mostly unskilled (bell-hops, cleaning crews, waiters, clerks, drivers), with little investment by the hotel owners in education or training.

Other firms with low costs of bankruptcy are retail stores selling to the general public, supermarkets, and department stores. The more specialized the store's products, the higher its cost of bankruptcy.

A higher debt-to-equity ratio increases the value of the debt tax shields, thereby increasing the value of the firm, but it also increases the probability of bankruptcy. If the cost of bankruptcy is high, the higher probability of bankruptcy soon outweighs the value of the tax shields.

Suppose the probability of bankruptcy is α^2, where α is the percentage of debt.

● If the firm has no debt, it has no chance of bankruptcy.
● If the firm has a 10% debt ratio, it has a 0.01 (or 1%) chance of bankruptcy.
● If the firm is financed 100% by debt, it has a 1.0 (or 100%) chance of bankruptcy.

Suppose the cost of bankruptcy is 100% of assets for pharmaceutical firms and 50% for hotels. The present value of the debt tax shield is 35% × α. We solve

~ Hotels: 35% × α = 50% × α^2 ⇒ α = 70%

~ Pharmaceutical firms: $35\% \times \alpha = 100\% \times \alpha^2 \Rrightarrow \alpha = 35\%$

*Question 1.4: Debt-to-Equity Ratio

Assume debt (interest) payments are from *pre-tax* funds and stockholder dividends are from *after-tax* funds (as is now the law), and the marginal tax rate is 35%. Firms in the hotel industry have debt-to-equity ratios of 25% (on average).

If the Congress changes the corporate tax rate to 15%, which of the following is true?

A.   The present value of the debt tax shields rises, and the average debt-to-equity ratio rises.
B.   The present value of the debt tax shields falls, and the average debt-to-equity ratio falls.
C.   The present value of the debt tax shields rises, and the average debt-to-equity ratio falls.
D.   The present value of the debt tax shields falls, and the average debt-to-equity ratio rises.
E.   The present value of the debt tax shields does not change, and the average debt-to-equity ratio falls.

Answer 1.4: B

The present value of the debt tax shields is the corporate tax rate times the market value of the debt. If the Congress raises the corporate tax rate, the present value of the debt tax shields increases. The equilibrium point where the present value of the debt tax shields equals the cost of bankruptcy is at a higher percentage of debt.

*Question 1.5: Beta of Assets

The corporate tax rate is 35%, the risk-free rate is 7%, and the market risk premium is 8%. A firm is at its optimal debt-to-equity ratio, and its

● equity has a CAPM beta of 100%
● debt has a CAPM beta of 12.5%
● assets have a CAPM beta of 75%

What is the firm's debt-to-equity ratio?

A.   20%
B.   25%
C.   40%
D.   50%
E.   75%

Answer 1.5: B

The weighted average cost of capital is $R_a = \alpha \times R_d \times (1 - \tau) + (1 - \alpha) \times R_e$, where α is the percentage of debt, R_a is the return on debt, R_d is the return on debt, and R_e is the return on equity. We solve for α as $\alpha = (R_a - R_e) / (R_d \times (1 - \tau) - R_e)$

● equity has a CAPM beta of 100% ⇒ the expected return = 7% + 100% × 8% = 15.00%
● debt has a CAPM beta of 12.5% ⇒ the expected return = 7% + 12.5% × 8% = 8.00%
● assets have a CAPM beta of 75% ⇒ the expected return = 7% + 75% × 8% = 13.00%

$$\alpha = (13\% - 15\%) / (8\% \times (1 - 35\%) - 15\%) = 20.41\% \approx 20\%.$$

The percentage of debt is 20%, so the debt-to-equity ratio is 20% / 80% = 25%.

Question: A candidate asks on the discussion forum why we can't use the weighted average of the debt and equity betas to get the asset beta.

Answer: Debt payments are tax deductible; stockholder dividends are from after-tax funds. We use a weighted average of the debt and equity betas, but we must first convert them to the same pre-tax or after-tax basis.