Corporate finance, Stocks, miscellaneous exam problems

(The attached PDF file has better formatting.)

*Question 1.1: Expected Return and Standard Deviation

An investor forms an equally value weighted portfolio from two stocks:

- Stock Y has an expected return of 12% and a standard deviation of 60%.
- Stock Z has an expected return of 15% and a standard deviation of 50%.

Which of the following is true?

A. The expected return on the portfolio is between 12% and 15%.
B. The standard deviation of the portfolio return is between 50% and 60%.
C. The variance of the portfolio return is between 25% and 36%.
D. The standard deviation of the portfolio return is greater than 55%.
E. The variance of the portfolio return is greater than 30.5%.

Answer 1.1: A

Statement A: The expected return on the portfolio is value weighted average of the expected returns on the stocks: ½ × (12% + 15%) = 13.5%.

Statements C and E: The variance of the portfolio depends on the correlation, which may range from –1 to +1.

The *minimum* variance occurs when the correlation is –1:

$$\tfrac{1}{2} \times \tfrac{1}{2} \times 60\%^2 + \tfrac{1}{2} \times \tfrac{1}{2} \times 50\%^2 + \tfrac{1}{2} \times \tfrac{1}{2} \times 2 \times 60\% \times 50\% \times -1 = 0.250\%$$

The *maximum* variance occurs when the correlation is +1:

$$\tfrac{1}{2} \times \tfrac{1}{2} \times 60\%^2 + \tfrac{1}{2} \times \tfrac{1}{2} \times 50\%^2 + \tfrac{1}{2} \times \tfrac{1}{2} \times 2 \times 60\% \times 50\% \times 1 = 30.250\%$$

Statements B and D: The standard deviation is the square root of the variance. It ranges from $0.0250\%^{1/2}$ = 5.00% to $30.250\%^{1/2}$ = 55.00%.

*Question 1.2: Expected Return and Standard Deviation

An investor forms a portfolio from two stocks.

- Stock Y has an expected return of 12% and a standard deviation of 60%.
- Stock Z has an expected return of 15% and a standard deviation of 50%.
- The correlation of returns between the two stocks is 40%.

If the portfolio has an *expected return* of 14%, what is its *standard deviation*?

A. 40%
B. 45%
C. 50%
D. 55%

E. 60%

Answer 1.2: B

The expected return of the portfolio is the weighted average of the expected returns on the two stocks. We work out the weights from the expected return of the portfolio:

We derive the weights from the expected return of the portfolio:

$$\alpha \times 12\% + (1 - \alpha) \times 15\% = 14\% \Rrightarrow \alpha \times (12\% - 15\%) = 14\% - 15\% \Rrightarrow \alpha = \tfrac{1}{3}$$

We work out the variance of the portfolio from the standard deviations and correlation. The variance of each stock is the square of its standard deviation and the covariance of the two stocks is the correlation times their standard deviations.

$$\text{variance} = \tfrac{1}{3}^2 \times 0.600^2 + \tfrac{2}{3}^2 \times 0.500^2 + 2 \times \tfrac{1}{3} \times \tfrac{2}{3} \times 0.4 \times 0.6 \times 0.5 = 0.204$$

The standard deviation of the portfolio is the square root of the variance:

$$0.204^{1/2} = 0.452 \approx 45\%.$$

*Question 1.3: Expected Return and Standard Deviation

An investor forms an equally weighted portfolio from two stocks:

- Stock Y has an expected return of 12% and a standard deviation of 60%.
- Stock Z has an expected return of 15% and a standard deviation of 50%.

If the standard deviation of the portfolio is 46.1%, what is the correlation of returns of the two stocks?

A. 20%
B. 25%
C. 30%
D. 35%
E. 40%

Answer 1.3: E

$$(0.50^2 \times 60\%^2 + 0.50^2 \times 50\%^2 + 2 \times \rho \times 0.50 \times 0.50 \times 60\% \times 50\%)^{1/2} = 46.1\% \Rrightarrow$$
$$\rho = [\ 0.461^2 - (0.50^2 \times 60\%^2 + 0.50^2 \times 50\%^2\ )\ ] \,/\, (2 \times 0.50 \times 0.50 \times 60\% \times 50\%) = 40.01\%$$

We verify as

$$(0.50^2 \times 60\%^2 + 0.50^2 \times 50\%^2 + 2 \times 40\% \times 0.50 \times 0.50 \times 60\% \times 50\%)^{1/2} = 46.1\%$$

*Question 1.4: Price Elasticity of Demand for Common Stocks

The price elasticity of demand is the percentage change in quantity demanded for a percentage change in the price. Which of the following is a reasonable price elasticity of demand for common stocks? Use the lesson of capital market efficiency that *seen one stock, seen 'em all*. Assume the stock market is *perfectly efficient*.

A. $+\infty$
B. +1
C. 0
D. –1
E. $-\infty$

Answer 1.4: E

The price elasticity of demand is the percentage change in the quantity demanded divided by the percentage change in the price. As the price increases, the quantity demanded decreases, so the price elasticity of demand is negative.

Capital markets are competitive. In a perfectly competitive market, as the price *increases* a small amount, the quantity demanded *decreases* a large amount. At the limit, the price elasticity of demand is $-100\% / +\varepsilon \rightarrow -\infty$.

*Question 1.5: Price Weighted vs Value Weighted Returns

A *price weighted average* uses the stock price (the price of a single share) as the weights; a *value weighted average* uses the market value as the weights, or the share price times the number of shares.

- Firm Y has *2 million* shares which *increase* from $80 to $88.
- Firm Z has *8 million* shares which *decrease* from $40 to $36.

Which of the following is true?

	Average Return	
	Price Weighted	*Value Weighted*
A	+3.33%	–3.33%
B	+3.33%	+3.33%
C	no change	no change
D	–3.33%	+3.33%
E	+3.33%	no change

Answer 1.5: A

For a price weighted average, the weights are the stock prices, not the market value of the firms. For a value weighted average, the weights are the market values of the firms.

- For the price weighted average, the weights are $80 vs $40, or ⅔ vs ⅓. The weighted return is ⅔ × +10% + ⅓ × –10% = 3.33%.
- For the value weighted average, the weights are $160 million vs $320 million, or ⅓ vs ⅔. The weighted return is ⅓ × +10% + ⅔ × –10% = –3.33%.

*Question 1.6: Equally Weighted Portfolio

An investor creates an equally weighted portfolio of the largest 3,000 stocks. The variance of each stock's returns is 70%, and the correlation among the returns on each pair of stocks is 50%. What is the standard deviation of the return on the portfolio?

A. 35%
B. 50%
C. 59%
D. 70%
E. 84%

Answer 1.6: C

The standard deviation of the return on the portfolio is the square root of the variance of the return on the portfolio. The variance of an equally weighted portfolio of *two stocks* is

$$½ \times ½ \times \sigma^2 + 2 \times ½ \times ½ \times \rho \times \sigma \times \sigma + ½ \times ½ \times \sigma^2 = ½ \times \sigma^2 + ½ \times \rho \times \sigma^2$$

The variance of an equally weighted portfolio of *N stocks* is

$$N \times (1/N \times 1/N \times \sigma^2) + 2 \times N(N-1)/2 \times 1/N \times 1/N \times \rho \times \sigma \times \sigma = 1/N \times \sigma^2 + (N-1)/N \times \rho \times \sigma^2$$

$$= [\text{Variance} + (N-1) \times \text{Covariance}] / N = \text{Variance} \times (1 + (N-1) \times \rho) / N$$

Each of the N stocks has a weight of 1/N and a variance of σ^2 / N^2.

~ If the N stocks are independent, $\rho = 0$ and the covariance is zero. We add the variances of the stocks to get $N \times \sigma^2 / N^2 = \sigma^2 / N$.
~ If the N stocks are perfectly correlated, $\rho = 1$ and the covariance equals the variance. The N stocks are like N shares of the same stock, so the variance of the portfolio is σ^2.
~ If the N stocks are partly correlated, we use the full formula.

The variance of the portfolio is [70% × (1 + 2,999 × 50%)] / 3,000 = 0.350

The standard deviation is $0.350^{½} = 0.592$

*Question 1.7: Equally Weighted Portfolio

An investor creates an equally weighted portfolio of the largest 3,000 stocks. The variance of each stock's returns is 60%, and the correlation among the returns on each pair of stocks is 45%. Because of hostilities in South-East Asia, the stock market becomes more volatile. The variance of each stock's returns changes to 68%, and the correlation among the returns on each pair of stocks changes to 40%.

What is the *percentage change* in the *standard deviation* of the return on the portfolio?

A. The standard deviation decreases by 6%
B. The standard deviation decreases by 3%
C. The standard deviation does not change materially
D. The standard deviation increases by 3%
E. The standard deviation increases by 6%

Answer 1.7: C

The variance of an equally weighted portfolio of *N stocks* is

$$N \times (1/N \times 1/N \times \sigma^2) + 2 \times N(N-1)/2 \times 1/N \times 1/N \times \rho \times \sigma \times \sigma = 1/N \times \sigma^2 + (N-1)/N \times \rho \times \sigma^2$$

$$= [\text{Variance} + (N-1) \times \text{Covariance}] / N = \text{Variance} \times (1 + (N-1) \times \rho) / N$$

The variance of the portfolio *before* the change is [60% × (1 + 2,999 × 45%)] / 3,000 = 0.270

The standard deviation is $0.27^{½} = 0.520 = 52\%$.

The variance of the portfolio *after* the change is [68% × (1 + 2,999 × 40%)] / 3,000 = 0.272

The standard deviation is $0.272^{½} = 0.522 \approx 52\%$.