Corporate finance, tax depreciation, practice exam problems
*Question 1.1: Tax Depreciation
An asset with a depreciation basis of $\$ 700,000$ and no salvage value is depreciated by the following schedule for tax purposes.

| $\frac{\text { Year }}{}$ |  |
| :---: | ---: |
| 1 | Percent |
| 2 | $30 \%$ |
| 3 | $25 \%$ |
| 4 | $20 \%$ |
| 5 | $15 \%$ |
|  | $10 \%$ |

The marginal tax rate is $35 \%$ and the pre-tax borrowing rate is $12 \%$. All cash flows occur at the end of the year. What is the total after-tax present value at time 0 of the tax shields?
A. $\$ 200,000$
B. $\$ 204,000$
C. $\$ 208,000$
D. $\$ 212,000$
E. \$216,000

Answer 1.1: B
The rate to compute the after-tax present value of the depreciation tax shields is the after-tax borrowing rate.

| Year | Percent | Depreciation | Tax Shield | Present Value |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $30 \%$ | $\$ 210,000$ | $\$ 73,500$ | $\$ 68,182$ |
| 2 | $25 \%$ | $\$ 175,000$ | $\$ 61,250$ | $\$ 52,707$ |
| 3 | $20 \%$ | $\$ 140,000$ | $\$ 49,000$ | $\$ 39,115$ |
| 4 | $15 \%$ | $\$ 105,000$ | $\$ 36,750$ | $\$ 27,213$ |
| 5 | $10 \%$ | $\$ 70,000$ | $\$ 24,500$ | $\$ 16,830$ |
| Total | $100 \%$ | $\$ 700,000$ | $\$ 245,000$ | $\$ 204,046$ |

The depreciation is the percentage given in the problem times the $\$ 700,000$ basis. The tax shield is $35 \%$ times the depreciation. The after-tax borrowing rate is $12 \% \times(1-35 \%)=7.80 \%$. The present value of the tax shield is the tax shield divided by 1.0780 raised to the power of the years.

The total present value of the five years' depreciation is $\$ 204,046$.
Question: What do we mean by the present value of the tax shields? And why do we use the after-tax rate?
Answer: The after-tax present value of the tax shields is the amount of cash receive now that has the same value as the tax shields received each year.

Suppose one firm receives $\$ 204,046$ now, and a second firm receives the tax shields each year in the future. To decide which firm has the better deal, we ask: if the first firm had to pay the cash of the tax shields, would it have positive or negative dollars left after 5 years?

We suppose first that the $12 \%$ borrowing rate is also the investment yield. The first firm invests the $\$ 204,046$ at $12 \%$.
~ The investment increases to $\$ 204,046 \times 1.12=\$ 228,532$ after one year. The firm pays taxes of $\$ 204,046 \times 12 \% \times 35 \%=\$ 8,570$, so it has $\$ 228,532-\$ 8,570=\$ 219,962$. It pays the $\$ 73,500$ tax shield, leaving it with $\$ 146,462$.
$\sim$ The investment increases to $\$ 146,462 \times 1.12=\$ 164,037$ after a second year. The firm pays taxes of $\$ 146,462 \times 12 \% \times 35 \%=\$ 6,151$, so it has $\$ 164,037-\$ 6,151=\$ 157,886$. It pays the $\$ 61,250$ tax shield, leaving it with $\$ 96,636$.

At the end of five years, the firm's investment just pays off the last tax shield.

## *Question 1.2: Tax Depreciation

A project requires an initial investment of $\$ 10$ million. The project lasts 10 years and has a salvage value of zero at the end of 10 years.

By law, the entire investment should be written off in year 0 , when the investment is made. The tax rate is $35 \%$. For a fee of $\mathrm{N} \times \$ 10,000$, a tax accountant can arrange for the investment to be depreciated over N years (pro-rata). What is the optimal fee paid to the tax accountant?
A. $\$ 0$ (write off the investment when it is made)
B. $\$ 10,000$ (depreciate the investment over 1 year)
C. $\$ 30,000$ (depreciate the investment over 3 years)
D. $\$ 50,000$ (depreciate the investment over 5 years)
E. $\$ 100,000$ (depreciate the investment over 10 years)

## Answer 1.2: A

Writing off the investment immediately maximizes after-tax income. Tax depreciation lowers after-tax income in this scenario.

