MS Module 2 Normal distribution practice exam questions.

(The attached PDF file has better formatting.)

A sample from a normal distribution has summary statistics:

- $n = 10$
- $\sum x_i = 21$
- $\sum x_i^2 = 159$

Question 2.1: Sample mean

What is the mean of the sample?

Answer 2.1: The mean of the sample is $21 / 10 = 2.1$

Question 2.2: Sample variance

What is the variance of the sample?

Answer 2.2: The sum of squared deviations is $159 – 21^2 / 10 = 114.90$ and the sample variance is

$$(159 – 21^2 / 10) / (10 – 1) = 12.767$$

Question 2.3: Standard deviation

What is the standard deviation of the sample?

Answer 2.3: The standard deviation is the square root of the variance: $((159 – 21^2 / 10) / (10 – 1))^{0.5} = 3.573$

Question 2.4: Maximum likelihood estimate of the variance

What is the maximum likelihood estimate of the variance?

Answer 2.4: $(159 – 21^2 / 10) / 10 = 11.490$

(The maximum likelihood estimate of the variance divides by N, not by (N-1).)

Question 2.5: Maximum likelihood estimate of the standard deviation

What is the maximum likelihood estimate of the standard deviation?

Answer 2.5: $((159 – 21^2 / 10) / 10)^{0.5} = 3.390$

Question 2.6: Standard error of the sample mean

What is the standard error of the sample mean?

Answer 2.6: $3.573 / 10^{0.5} = 1.130$

(Standard error of the mean = standard deviation of the sample / square root of the number of observations)

Question 2.7: Confidence interval, lower bound

What is the lower bound of the 90% two-sided confidence interval for the mean of the normal distribution?

Answer 2.7: 2.100 – 1.645 × 1.130 = 0.241

(For a confidence level of 90%, α = 10%, and $z_{\alpha/2}$ = 1.645 (table look-up). The lower bound of the two-sided confidence interval for the mean of the normal distribution = mean – $z_{\alpha/2}$ × standard error of the mean.)

Question 2.8: Confidence interval, upper bound

What is the upper bound of the 90% two-sided confidence interval for the mean of the normal distribution?

Answer 2.8: 2.100 + 1.645 × 1.130 = 3.959