BM Ch9 optimal portfolio practice exam question

The market has only two risky securities, with expected returns, standard deviations, and market values of

	Expected Return	*Standard Deviation*	*Market Value*
Stock Y	9.19%	30.40%	45.59 million
Stock Z	13.80%	67.20%	18.97 million

The correlation of stocks Y and Z is 54.62%. Risk-free bonds yield 4.74%. An investor who can borrow or lend at the risk-free rate forms an optimal portfolio of risk-free bonds and risky securities.

Question 8.1: Expected return of market portfolio of risky assets

What is the expected return of the market portfolio of risky securities?

Answer 8.1: The expected return of the market portfolio of risky securities is a weighted average by market value:

$$((9.19\%) \times 45.59 + (13.80\%) \times 18.97) / (45.59 + 18.97) = 10.54\%$$

Question 8.2: Variance of market portfolio of risky assets

What is the variance of the market portfolio of risky securities?

Answer 8.2: The variance of the market portfolio of risky securities is

$$((30.40\%)^2 \times 45.59^2 + (67.20\%)^2 \times 18.97^2 + 2 \times (54.62\%) \times (30.40\%) \times 45.59 \times (67.20\%) \times 18.97) / (45.59 + 18.97)^2 = 13.14\%$$

Question 8.3: Standard deviation of market portfolio of risky assets

What is the standard deviation of the market portfolio of risky securities?

Answer 8.3: The standard deviation of the market portfolio of risky securities is the square root of the variance

$$(13.14\%)^{0.5} = 36.25\%$$

Question 8.4: Composition of optimal portfolio

What is the composition of the optimal portfolio with a standard deviation of 42.7%?

Answer 8.4: The optimal portfolio is a combination of risk-free bonds with a standard deviation of zero and the market portfolio of risky securities that have a standard deviation of 36.25%. For a standard deviation of 42.7%, we solve

$$36.25\% \times Z + 0 \times (1 - Z) = 42.7\%$$
$$\Rightarrow Z = (42.7\%) / (36.25\%) = 117.79\%$$
$$\Rightarrow (1 - Z) = 1 - 117.79\% = -17.79\%$$

For an optimal portfolio with a standard deviation of 42.7% and a market value of 100, the investor *sells* 17.79 of risk-free bonds with a standard deviation of zero and buys 117.79 of the market portfolio of risky securities with a standard deviation of 36.25%. This investor wants a portfolio even riskier than the market portfolio.

Question 8.5: Expected return of optimal portfolio

What is the expected return on an optimal portfolio with a standard deviation of 42.7%?

The expected return on the optimal portfolio with a standard deviation of 42.7% is a weighted average of its two parts:

$$117.79\% \times 10.54\% + -17.79\% \times 4.74\% = 11.57\%$$

Question 8.6: Composition of optimal portfolio

What is the composition of the optimal portfolio with an expected return of 6.24%?

Answer 8.6: The optimal portfolio is a combination of risk-free bonds with an expected return of 4.74% and the market portfolio of risky securities that have an expected return of 10.54%. For an expected return of 6.24%, we solve

$$10.54\% \times Z + 4.74\% \times (1 - Z) = 6.24\%$$

$$\Rrightarrow Z = (6.24\% - 4.74\%) / (10.54\% - 4.74\%) = 25.86\%$$

$$\Rrightarrow (1 - Z) = 1 - 25.86\% = 74.14\%$$

For an optimal portfolio with an expected return of 6.24% and a market value of 100, the investor

- buys 25.86 of the market portfolio of risky securities with an expected return of 10.54% and
- buys 74.14 of risk-free bonds with an expected return of 4.74%.

Question 8.7: Standard deviation of optimal portfolio

What is the standard deviation on an optimal portfolio with an expected return of 6.24%?

Answer 8.7: The risk-free bonds have a standard deviation of zero and are not correlated with the market portfolio of risky securities, which have a standard deviation of 36.25%. This optimal portfolio has a standard deviation of 25.86% × 36.25% = 9.37%.