

Microeconomics Mod 4 elasticity practice exam questions

(The attached PDF file has better formatting.)

A monopolist faces a linear demand curve, produces the monopoly quantity, and sets monopoly prices.

- At price $P = 4.58$, total revenue is 254.469
- At price $P = 8.86$, total revenue is 319.731

The demand curve is linear: $Q = \alpha - \beta \times P$. By convention, this posting uses a positive value for β with the negative sign shown separately.

Question 4.1: Linear demand curve

What are the values of the α and β parameters of the linear demand curve?

Answer 4.1: Total revenue is price \times quantity, giving

- $4.58 \times (\alpha - \beta \times 4.58) = 254.469$
- $8.86 \times (\alpha - \beta \times 8.86) = 319.731$

Solving the simultaneous linear equations gives

$$8.86 \times 4.58 \times (\alpha - \beta \times 4.58) = 8.86 \times 254.469$$

$$4.58 \times 8.86 \times (\alpha - \beta \times 8.86) = 4.58 \times 319.731$$

using the values

$$8.86 \times 4.58 = 40.579$$

$$8.86 \times 8.86 \times 4.58 = 359.528$$

$$8.86 \times 4.58 \times 4.58 = 185.851$$

$$40.579 \alpha - 185.851 \beta = 8.86 \times 254.469 = 2,254.595$$

$$40.579 \alpha - 359.528 \beta = 4.58 \times 319.731 = 1,464.368$$

$$\beta = (2,254.595 - 1,464.368) / (359.528 - 185.851) = 4.550$$

$$\alpha = (185.851 \beta + 2,254.595) / 40.579 = (185.851 \times 4.550 + 2,254.595) / 40.579 = 76.400$$

The demand curve is $Q = 76.400 - 4.550 \times P$.

Question 4.2: Price, quantity, and total revenue

What is the *quantity* at price $P = 12.82$?

Answer 4.2: We derive the quantity from the demand curve:

$$Q = 76.400 - 4.550 \times 12.82 = 18.069$$

Question 4.3: Price elasticity of demand

What is the price elasticity of demand at price $P = 12.82$?

Answer 4.3: $\eta =$ the price elasticity of demand $= \partial Q / \partial P \times P / Q = -\beta \times P / Q = -4.550 \times 12.82 / 18.069 = -3.228$

Question 4.4: Marginal revenue and price

What is the *marginal revenue* at price $P = 12.82$?

Answer 4.4: Use the relation $MR = P \times (1 - 1 / |\eta|)$ to give $12.82 \times (1 - 1/3.228) = 8.849$

Question 4.5: Marginal revenue and quantity

At *what quantity* is the marginal revenue equal to zero?

Answer 4.5: For a linear demand curve, the marginal revenue curve is twice as steep as the demand curve, so marginal revenue equals zero half-way between $Q = 0$ and $Q = \alpha$.

$$\frac{1}{2} \times \alpha = \frac{1}{2} \times 76.400 = 38.200$$

Question 4.6: Price and elasticity

At *what price* is the price elasticity of demand equal to -1.30?

Answer 4.6: Solve for price as a function of the elasticity of demand and the parameters of the demand curve:

$\eta =$ the price elasticity of demand $= \partial Q / \partial P \times P / Q = -\beta \times P / (\alpha - \beta \times P)$.

$$1/\eta = -(\alpha - \beta \times P) / (\beta \times P) = 1 - \alpha / (\beta \times P)$$

$$1 - 1/\eta = \alpha / (\beta \times P)$$

$$P = (\alpha / \beta) \times \eta / (\eta - 1) = (76.400 / 4.550) \times -1.30 / (-1.30 - 1) = (76.400 / 4.550) \times 1.30 / (1 + 1.30) = 9.491$$