

MS Module 23: Actuarial risk classification – practice problems

(The attached PDF file has better formatting.)

Reading on discussion forum: Actuarial risk classification

Exercise 23.1: Balance principle multiplicative model

The mean values and the number of observations in each cell of a 2×2 classification table are

<i>Means</i>	<i>Column 1</i>	<i>Column 2</i>	<i>Observations</i>	<i>Column 1</i>	<i>Column 2</i>
<i>Row 1</i>	50	30	<i>Row 1</i>	15	12
<i>Row 2</i>	20	8	<i>Row 2</i>	6	10

Illustration: The cell in row 1 column 1 has a mean of 50 from a sample of 15 observations.

An actuary is setting class relativities for insurance pricing using a multiplicative model balance principle, with

- ! a base rate of 10
- ! a starting relativity for column 1 of 1.8
- ! a starting relativity for column 2 of 1.0

We use the following notation:

- B = base rate
- r_1 = relativity for Row 1
- r_2 = relativity for Row 2
- c_1 = relativity for Column 1
- c_2 = relativity for Column 2

- A. What are the observed totals for each cell, row, and column?
- B. What are the formulas for each cell, row, and column using base rates and relativities?
- C. What is the equation to balance along Row 1?
- D. What is the implied relativity for Row 1, given the starting relativities by column?
- E. What is the equation to balance along Row 2?
- F. What is the implied relativity for Row 2, given the starting relativities by column?
- G. What is the equation to balance down Column 1?
- H. What is the implied relativity for Column 1, given the computed relativities by row?
- I. What is the equation to balance down Column 2?
- J. What is the implied relativity for Column 2, given the computed relativities by row?

Part A: The observed totals by cell are

- ! Row 1, Column 1: $50 \times 15 = 750$
- ! Row 1, Column 2: $30 \times 12 = 360$
- ! Row 2, Column 1: $20 \times 6 = 120$
- ! Row 2, Column 2: $8 \times 10 = 80$

The table below shows the totals by row and by column:

	Column 1	Column 2	Total
Row 1	750	360	1,110
Row 2	120	80	200
Total	870	440	1,310

Part B: The formulas for the mean values by cell are

- ! Row 1, Column 1: $B \times r_1 \times c_1$
- ! Row 1, Column 2: $B \times r_1 \times c_2$
- ! Row 2, Column 1: $B \times r_2 \times c_1$
- ! Row 2, Column 2: $B \times r_2 \times c_2$

Using $obss_{j,k}$ as the number of observations in Row j and Column k , the totals by cell are

- ! Row 1, Column 1: $B \times r_1 \times c_1 \times obss_{1,1}$
- ! Row 1, Column 2: $B \times r_1 \times c_2 \times obss_{1,2}$
- ! Row 2, Column 1: $B \times r_2 \times c_1 \times obss_{2,1}$
- ! Row 2, Column 2: $B \times r_2 \times c_2 \times obss_{2,2}$

We add the expressions above for the totals by row and by column:

- ! Row 1: $B \times r_1 \times (c_1 \times obss_{1,1} + c_2 \times obss_{1,2})$
- ! Row 2: $B \times r_2 \times (c_1 \times obss_{2,1} + c_2 \times obss_{2,2})$
- ! Column 1: $B \times (r_1 \times obss_{1,1} + r_2 \times obss_{2,1}) \times c_1$
- ! Column 2: $B \times (r_1 \times obss_{1,2} + r_2 \times obss_{2,2}) \times c_2$

Part C: Using the formula for the Row 1 relativity, the base rate of 10, and the starting relativities of 1.8 for Column 1 and 1.0 for Column 2, we balance the observed and theoretical values to give

$$750 + 360 = 10 \times r_1 \times (1.80 \times 15 + 1.00 \times 12)$$

Part D: The implied relativity for Row 1 is

$$r_1 = (750 + 360) / (10 \times (1.80 \times 15 + 1.00 \times 12)) = 2.846154$$

Part E: Using the formula for the Row 2 relativity, the base rate of 10, and the starting relativities of 1.8 for Column 1 and 1.0 for Column 2, we balance the observed and theoretical values to give

$$120 + 80 = 10 \times r_2 \times (1.80 \times 6 + 1.00 \times 10)$$

Part F: The implied relativity for Row 2 is

$$r_2 = (120 + 80) / (10 \times (1.80 \times 6 + 1.00 \times 10)) = 0.961538$$

Part G: Using the formula for the Column 1 relativity, the base rate of 10, and the implied relativities of 2.846154 for Row 1 and 0.961538 for Row 2, we balance the observed and theoretical values to give

$$750 + 120 = 10 \times (2.846154 \times 15 + 0.961538 \times 6) \times c_1$$

Part H: The implied relativity for Column 1 is

$$c_1 = (750 + 120) / (10 \times (2.846154 \times 15 + 0.961538 \times 6)) = 1.795238$$

Part I: Using the formula for the Column 2 relativity, the base rate of 10, and the implied relativities of 2.846154 for Row 1 and 0.961538 for Row 2, we balance the observed and theoretical values to give

$$360 + 80 = 10 \times (2.846154 \times 12 + 0.961538 \times 10) \times c_1$$

Part J: The implied relativity for Column 1 is

$$c_2 = (360 + 80) / (10 \times (2.846154 \times 12 + 0.961538 \times 10)) = 1.005272$$