MS Module 19: Correlation – practice problems

(The attached PDF file has better formatting.)

Exercise 19.1: Correlation

A regression model $Y_j = \beta_0 + \beta_1 X_j + \epsilon_j$ has N = 11 observations. The sample correlation between X and Y is 0.60. We test the null hypothesis H_0: $\rho = 0$ (the true correlation between the X and Y variables is zero).

A. What is the *t* value to test the null hypothesis?
B. What is the *p* value to test the null hypothesis?

Part A: The *t* value is $R \sqrt{(n-2)} / \sqrt{(1 - R^2)} = 0.6 \times (11 - 2)^{0.5} / (1 - 0.6^2)^{0.5} = 2.25000$

Part B: The *t* distribution has n-2 degrees of freedom, so the *p* value for a two-tailed test is 0.051 (table look-up or spread-sheet function).

Question: The sample correlation is 0.60, which is much different from zero, yet the *p* value is 5.1%, which does not satisfy even a 5% significance level.

Answer: The scenario has only 11 observations. Even if the true correlation is zero, a sample with only a few observations often shows a high sample correlation.

Exercise 19.2: Correlation and β_1

A linear regression with 11 data points has an estimated β_1 of 4.5 and a sample correlation between the X and Y values of 0.60.

A. What is the *t* value to test the null hypothesis that the correlation ρ is zero?
B. What is the *t* value to test the null hypothesis that β_1 is zero?
C. What is the standard deviation of the estimate of β_1?

Part A: The *t* value to test the null hypothesis that the correlation ρ is zero is $r\sqrt{(n-2)} / \sqrt{(1-r^2)} =$

$$0.6 \times (11 - 2)^{0.5} / (1 - 0.6^2)^{0.5} = 2.25000$$

Part B: The *t* value to test the null hypothesis that β_1 is zero is $\hat{\beta}_1 / \sigma(\hat{\beta}_1) = 4.5 / \sigma(\hat{\beta}_1)$, where

- $\hat{\beta}_1$ is the estimate of β_1.
- $s(\hat{\beta}_1)$ is the standard deviation of the estimate of β_1.

Part C: The two tests are the same: $\rho = 0$ implies $\beta_1 = 0$ and vice versa.

$$4.5 / s(\hat{\beta}_1) = 2.25 \Rightarrow s(\hat{\beta}_1) = 4.5 / 2.25 = 2.$$