

MS Module 17: Regression analysis confidence intervals and hypothesis testing (overview 2<sup>nd</sup> edition)

(The attached PDF file has better formatting.)

(Readings from the second 2<sup>nd</sup> edition of the Devore text.)

Reading: §12.3 Inferences about the regression coefficient  $\beta_1$

Distinguish between parameters and their estimates.

$\beta_1$  is a parameter of the regression equation (its slope); it has a fixed but unknown value, with no standard deviation. The estimate of  $\beta_1$  (shown as  $\hat{\beta}_1$  in the textbook) is a sample statistic with a sampling distribution. Inferences from regression analysis depend on the attributes of this sample statistic (such as its standard deviation) and of other sample statistics, such as the estimate of  $\sigma^2$ .

Know the degrees of freedom for each statistical test. Most problems in previous modules subtract 1 from the number of observations or the number of classes. A regression equation has two parameters, so the degrees of freedom is the number of observations minus 2. The estimator for  $\sigma$  divides by  $n-2$  (see equations below Figure 12.15) and the T-ratio has  $n-2$  degrees of freedom. Multiple regression has more parameters, and the degrees of freedom decrease accordingly.

$\beta_1$  can be expressed as a linear combination the observed Y values, where the coefficients in this combination are functions of the X values. Know the formula for these coefficients, which appears in confidence intervals, prediction intervals, and standardized residuals as well.

Final exam problems test three types of intervals for regression analysis:

- ! The confidence interval for  $\beta_1$ , which is a range with upper and lower bounds.
- ! The confidence interval for  $\hat{y}$  (the fitted value), which depends on the  $x$  value.
- ! The prediction interval, part of which depends on the  $x$  value and part of which is constant.

This module computes the confidence interval for  $\beta_1$ ,  $t$  values to test hypotheses about  $\beta_1$ , and the  $p$  values to estimate the probability of a Type I error. The next module derives the other two intervals. All three intervals are tested on the final exam.

Distinguish between the standard deviation of the error term ( $\sigma$ ) and the standard deviation of the estimate of  $\beta_1$ . The standard deviation of the estimate of  $\beta_1 = s / \sqrt{S_{xx}}$ . A common error is to forget to divide by  $\sqrt{S_{xx}}$  when testing hypotheses about  $\beta_1$ .

Read the section "A Confidence Interval for  $\beta_1$ ." Note the degrees of freedom and the standard error. Example 12.12 shows how to compute the confidence interval for  $\beta_1$  from the summary statistics.

Read the section "Regression and ANOVA." Know the relation of the  $t$  value for hypothesis testing and the  $F$  test for the regression analysis ( $t^2 = f$ ); see the text right below Table 12.2.

Skip the section "Fitting the Logistic Regression Model." The final exam does not cover maximum likelihood fitting.

Review end of chapter exercises 31 a and b, 33, 34 a and b, 35 a and b.