MS Module 15 or 21: Logistic regression models – practice problems

(The attached PDF file has better formatting.)

[Logistic regression is in module 15 for the 2nd edition of the text and module 21for the 3rd edition of the text. The text reading is the same in the two editions. The 3rd edition of the text has additional material on logistic regression that is not on the syllabus for this course.]

Exercise 15.1: Logistic regression

A probability Y is related to the independent variable X by logistic regression:

$$Y = p(x) = \exp(\beta_0 + \beta_1 x) / (1 + \exp(\beta_0 + \beta_1 x))$$

- When X = 7, the probability Y is 20%.
- When X = 8, the probability Y is 25%.

A. At X = 7, what is the odds ratio of Y?
B. At X = 8, what is the odds ratio of Y?
C. At X = 11, what is the odds ratio of Y?
D. At X = 11, what is the probability of Y?

Part A: The odds ratio of Y at x = 7 is 20% / (1 – 20%) = 0.2500.

Part B: The odds ratio of Y at x = 8 is 25% / (1 – 25%) = 0.3333.

Part C: The slope parameter β_1 is the change in the *log odds* for a 1-unit increase in x, so the odds ratio itself changes by the multiplicative factor $\exp(\beta_1)$ when x increases by 1 unit. This factor is

$$0.333333 / 0.25 = 1.33333$$

11 is 3 units more than 8, so the odds ratio of Y at x = 11 is $0.333333 \times 1.333333^3 = 0.79012$

Part D: If Y = the probability and R = the odds ratio, $R = Y / (1–Y) \Rightarrow Y = R / (1+R)$.

The probability is the odds ratio / (1 + odds ratio), so the probability of Y at x = 11 is

$$0.79012 / 1.79012 = 44.14\%$$

Exercise 15.2: Logistic regression

A statistician uses a logistic regression model:

! The independent variable X is a quantitative predictor.
! The dependent variable Y is 1 if the observation is a success and 0 otherwise.

The estimate of β_1 is –0.20.

The odds of success at X = 1 are 50%.

A. What is the probability of success at X = 1?
B. What are the odds of success at X = 3?
C. What is the probability of success at X = 3?
D. What are the odds of success at X = 0?
E. What is the probability of success at X = 0?
F. What is β_0?

Part A: If the probability of success is P, the odds of success are P/(1-P).

Given that P/(1-P) = 50%, P = ½ – ½P ⇛ P = a.

The formula is probability = odds / (1 + odds) = 50% / (1 + 50%) = 0.3333

Part B: For each one unit increase in X, the odds of success increase by a factor exp($\hat{\beta}_1$) = $e^{-0.20}$ = 0.81873

3 is 2 units more than 1, so the odds of success at X = 3 are 50% × 0.81873^2 = 0.335159

Part C: P/(1-P) = 0.33516 ⇛ 1.33516 P = 0.33516 ⇛ P = 0.33516 / 1.33516 = 0.25103

Part D: 0 is 1 unit less than 1, so the odds of success at X = 0 are 50% / 0.81873 = 0.610702

Part E: The probability of success at X = 0 is 0.610702 / 1.610702 = 0.379153

Part F: For logistic regression, Y = exp(β_0) / (1 + exp(β_0 + β_1 × X)).

If Y = 0.379153 at X = 0, then exp(β_0) / (1 + exp(β_0)) = 0.379153 ⇛

(1 – 0.379153) × exp(β_0) = 0.379153 ⇛

exp(β_0) = 0.379153 / (1 – 0.379153) = 0.610703

β_0 = *ln*(0.610703) = –0.49314