

MS Module 4: Hypotheses and Test Procedures (overview 3<sup>rd</sup> edition)

(The attached PDF file has better formatting.)

(Readings from the third 3<sup>rd</sup> edition of the Devore, Berk, and Carlton text.)

[Two complex statistical procedures that are not used in actuarial work (old module 10, “Levene’s test” and old module 11, “Tukey’s procedure”) have been removed from the syllabus, simplifying modules 10-14 on analysis of variance (ANOVA). To keep the 24 module sequence,

- ! Module 4 Hypotheses and Test Procedures is now split into
  - " Module 4 “Hypotheses and Test Procedures” (covering Type 1 and Type 2 errors)
  - " Module 5 “Tests about a population mean”
  
- ! Module 5 Hypothesis testing of proportions is now split into
  - " Module 6 “Tests About a Population Proportion”
  - " Module 7 “Hypothesis testing –  $p$  values”

The old modules 6-9 have been renamed modules 8-11.]

Reading: §9.1: Hypotheses and Test Procedures

Classical mathematical statistics deals with hypothesis testing and confidence intervals. Final exam problems combine these subjects with the scenarios covered in later modules.

The null hypothesis may be simple (that a parameter equals some value, such as zero) or complex (that the parameter is more or less than a value). The alternative hypothesis is the negation of the null hypothesis: that the parameter does not equal the value or that it is less than or more than the value.

Understand Type 1 and Type 2 errors:  $\alpha$  denotes the probability of a Type I error and  $\beta$  denotes the probability of a Type II error. These probabilities depend on the rejection region, which depends on the type of null hypothesis and the confidence level. Final exam problems give either rejection regions or confidence levels to compute  $\alpha$ . They give also the true value of the parameter to compute  $\beta$ . Be sure you understand the line in the textbook: “In contrast to  $\alpha$ , there is not a single  $\beta$ . Instead, there is a different  $\beta$  for each different  $p$  less than .2. Thus there is a value of  $\beta$  for  $p = .15$  [in which case  $X \sim \text{Bin}(25, .15)$ ], another value of  $\beta$  for  $p = .1$ , and so on.”

For clarity, the textbook uses examples from a binomial distribution, so you can solve the arithmetic by pencil and paper. Most final exam problems use the normal distribution or a transformation of the normal distribution.

Confidence intervals and hypothesis testing have several forms:

- ! if the population is normally distributed with a known variance, use  $z$  values
- ! if the population is normally distributed with an unknown variance, use  $t$  values
  - " if the sample is large, the  $t$  value approximates the  $z$  value
- ! if the population is not normally distributed but the sample is large, the central limit theorem gives an approximate solution using  $t$  values

Review examples 9.5 and 9.6. Figure 9.1 panels b & c show why  $\beta$  differs depending on the true value of  $p$ .

Review end of chapter exercises 4, 5, 6, 7, 10, 11, 14 a and b.

