MS Module 21 Multiple regression s^2 adjusted R^2 practice exam questions

(The attached PDF file has better formatting.)

[The practice problems in the 24 modules explain the statistical procedures; the practice exam questions in this thread shows what you will be asked on the final exam.]

A multiple regression analysis $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$, with 5 data points and independent variables X_1 and X_2 has the following actual values (y_i) and fitted values ($\hat{y}_i$):

Actual Value	2.4	0.8	6.1	10.9	9.8
Fitted Value	2	4.0	6	8	10

! The null hypothesis is H_0: $\beta_1 = \beta_2 = 0$
! The alternative hypothesis is H_a: $\beta_1 \neq 0$ or $\beta_2 \neq 0$

Question 21.1: Residuals

What are the residuals for the five data points?

Answer 21.1: residual = actual value – fitted value:

obs	*fitted*	*actual*	*residual*	*SST*	*SSE*
#1	2	2.4	0.4	12.96	0.16
#2	4	0.8	-3.2	27.04	10.24
#3	6	6.1	0.1	0.01	0.01
#4	8	10.9	2.9	24.01	8.41
#5	10	9.8	-0.2	14.44	0.04
avg	6	6	0	78.46	18.86

Question 21.2: Total sum of squares

What is the total sum of squares (SST)?

Answer 21.2: average y-value = (2.4 + 0.8 + 6.1 + 10.9 + 9.8) / 5 = 6

$SST = (2.4 – 6)^2 + (0.8 – 6)^2 + (6.1 – 6)^2 + (10.9 – 6)^2 + (9.8 – 6)^2 = 78.46$

Question 21.3: Error sum of squares

What is the error sum of squares (SSE)?

Answer 21.3: $SSE = (2.4 – 2)^2 + (0.8 – 4)^2 + (6.1 – 6)^2 + (10.9 – 8)^2 + (9.8 – 10)^2 = 18.86$

Question 21.4: Least squares estimate for σ^2

What is s^2, the least squares estimate for σ^2?

Answer 21.4: 18.86 / (5 – 2 – 1) = 9.43

(least squares estimate for σ^2 = error sum of squares / degrees of freedom, which are N-k-1)

Question 21.5: R^2

What is R^2?

Answer 21.5: 1 – 18.86 / 78.46 = 75.96%

(R^2 = 1 – error sum of squares / total sum of squares)

Question 21.6: Adjusted R^2

What is the adjusted R^2?

Answer 21.6: 1 – 18.86 / (5 – 2 – 1) / (78.46 / (5 – 1)) = 51.92%

(adjust SSE and SST by their degrees of freedom: adjusted R^2 = 1 – MSE / MST
= 1 – [SSE / (n - (k + 1)] / [SST / (n - 1)])

Question 21.7: *F* value

What is the test statistic value *f* to test the null hypothesis?

Answer 21.7: ((78.46 – 18.86) / 2) / (18.86 / (5 – 2 – 1)) = 3.160

(test statistic *f* = [R^2 / k] / [(1 – R^2) / (n - (k + 1))])