MS Module 15 Logistic regression probability of success practice exam questions
[Module 15 for the $2^{\text {nd }}$ edition of the textbook and Module 21 for the $3^{\text {rd }}$ edition of the textbook.]
(The attached PDF file has better formatting.)
[The practice problems in the 24 modules explain the statistical procedures; the practice exam questions in this thread shows what you will be asked on the final exam.]

A statistician uses a logistic regression model:
! The independent variable $X$ is a quantitative predictor.
! The dependent variable Y is 1 if the observation is a success and 0 otherwise.
The estimate of $\beta_{1}$ is -0.21
The odds of success at $\mathrm{X}=1.8$ are 0.4493

Question 15.1: Probability of success
What is the probability of success at $X=1.8$ ?
Answer 15.1: $0.4493 /(1+0.4493)=31.00 \%$
(probability $=$ odds ratio / ( 1 + odds ratio)

Question 15.2: Multiplicative change in the odds ratio
What is the multiplicative change in the odds ratio when x increases by 1 unit?
Answer 15.2: $\mathrm{e}^{-0.21}=0.8106$
(the multiplicative change in the odds ratio when x increases by 1 unit $=\exp \left(\beta_{1}\right)$

Question 15.3: Odds of success
What are the odds of success at $X=2.7$ ?
Answer 15.3: $0.4493 \times 0.8106^{(2.7-1.8)}=0.3719$
(odds ratio at point $X_{3}=$ odds ratio at point $X_{2} \times$ (multiplicative change in odds ratio) ${ }^{\text {(value of point } X_{3} \text { - value of point } X_{2} \text { ) }}$

Question 15.4: Probability of success
What is the probability of success at $X=2.7$ ?
Answer 15.4: $0.3719 /(1+0.3719)=27.11 \%$
(probability $=$ odds ratio $/(1+$ odds ratio $)$

Question 15.5: Odds of success
What are the odds of success at $X=0$ ?
Answer 15.5: $0.4493 \times 0.8106^{(0-1.8)}=0.6557$
(odds ratio at point $\mathrm{X}_{3}=$ odds ratio at point $\mathrm{X}_{2} \times$ (multiplicative change in odds ratio) ${ }^{\text {(value of point } \times 3 \text { - value of point } \mathrm{X}_{2} \text { ) }}$

Question 15.6: Probability of success
What is the probability of success at $X=0$ ?
Answer 15.6: $0.6557 /(1+0.6557)=39.60 \%$
(probability $=$ odds ratio / ( 1 + odds ratio)

Question 15.7: $\beta_{0}$
What is $\beta_{0}$ ?
Answer 15.7: $\ln (0.6557)=-0.4221$
$\left(\beta_{0}=\ln (\right.$ odd ratio at $X=0)$ )

