MS Module 14 Two-factor ANOVA interaction practice exam questions

(The attached PDF file has better formatting.)

[The practice problems in the 24 modules explain the statistical procedures; the practice exam questions in this thread shows what you will be asked on the final exam.]

A two-factor classification table has two rows, two columns, and two observations in each cell.

| | Column 1 | Column 2 |
|---|---|---|
| Row 1 | 32; 33 | 32; 27 |
| Row 2 | 14; 29 | 20; 22 |

We use analysis of variance to test

- whether the Row 1 mean differs from the Row 2 mean
  - the null hypothesis is that the row means are equal
- whether the Column 1 mean differs from the Column 2 mean
  - the null hypothesis is that the column means are equal
- whether the interaction effects are significant
  - the null hypothesis is that the interaction effects are zero

Question 14.1: Square of sum of observations

What is the square of the sum of all the observations, or $x_{...2}$ ?

Answer 14.1: $(32 + 33 + 32 + 27 + 14 + 29 + 20 + 22)^2 = 43{,}681$

Question 14.2: Correction factor

What is the correction factor used for the total sum of squares and the treatment sums of squares (for both rows and columns)?

Answer 14.2: 43,681 / 8 = 5,460.125

(correction factor = the square of the sum of the observations / the number of observations)

Question 14.3: Sum of squares of observations

What is the sum of the squares of all the observations, or $\Sigma_i \Sigma_j \Sigma_k x_{ijk2}$ ?

Answer 14.3: $(32^2 + 33^2 + 32^2 + 27^2 + 14^2 + 29^2 + 20^2 + 22^2) = 5{,}787$

Question 14.4: Sum of squares of totals by cell

What is the sum of the squares of the totals in each cell, or $\Sigma_i \Sigma_j x_{ij}^2$ ?

Answer 14.4: $(32 + 33)^2 + (32 + 27)^2 + (14 + 29)^2 + (20 + 22)^2 = 11{,}319$

Question 14.5: Sum of squares of row totals

What is the sum of the squares of the row totals, or $\sum_j x_{i..2}$

Answer 14.5: $(32 + 33 + 32 + 27)^2 + (14 + 29 + 20 + 22)^2 = 22{,}601$

Question 14.6: Sum of squares of column totals

What is the sum of the squares of the column totals, or $\sum_j x_{.j.2}$

Answer 14.6: $(32 + 33 + 14 + 29)^2 + (32 + 27 + 20 + 22)^2 = 21{,}865$

Question 14.7: Total sum of squares

What is SST, the total sum of squared deviations?

Answer 14.7: 5,787 – 5,460.125 = 326.875

(total sum of squares = the sum of the squares of all the observations – the correction factor)

Question 14.8: SSA

What is SSA, the sum of squared deviations for the *i* dimension (the rows)?

Answer 14.8: 22,601 / 4 – 5,460.125 = 190.125

(SSA = the sum of the squares of the row totals / observations per row – the correction factor)

Question 14.9: SSB

What is SSB, the sum of squared deviations for the *j* dimension (the columns)?

Answer 14.9: 21,865 / 4 – 5,460.125 = 6.125

(SSB = the sum of the squares of the column totals / observations per column – the correction factor)

Question 14.10: Error sum of squares

What is SSE, the error sum of squared deviations?

Answer 14.10: 5,787 – 11,319 / 2 = 127.50

(error sum of squares = the sum of the squares of the observations – the sum of the squares of the totals in each cell / number of observations by cell)

Question 14.11: SSAB

What is SSAB, the sum of squared deviations for the interaction?

Answer 14.11: 326.875 – 190.125 – 6.125 – 127.50 = 3.125

Question 14.12: Degrees of freedom

What are the degrees of freedom for the rows (SSA)?

Answer 14.12: 2 – 1 = 1

(the degrees of freedom for the rows = number of rows – 1)

Question 14.13: Degrees of freedom

What are the degrees of freedom for the columns (SSB)?

Answer 14.13: 2 – 1 = 1

(the degrees of freedom for the columns = number of columns – 1)

Question 14.14: Degrees of freedom

What are the degrees of freedom for the interaction effects (SSAB)?

Answer 14.14: (2 – 1) × (2 – 1) = 1

(the degrees of freedom for the interaction effects = (number of rows – 1) × (number of columns – 1)

Question 14.15: Degrees of freedom

What are the degrees of freedom for the total sum of squares (SST)?

Answer 14.15: 8 – 1 = 7

(the degrees of freedom for the total sum of squares = number of observations – 1)

Question 14.16: Degrees of freedom

What are the degrees of freedom for the error sum of squares (SSE)?

Answer 14.16: 7 – 1 – 1 – 1 = 4

(degrees of freedom for SSE = degrees of freedom for SST – degrees of freedom for SSA – degrees of freedom for SSB – degrees of freedom for SSAB)

Question 14.17: Mean squared deviation for the rows

What is MSA, the mean squared deviation for the rows?

Answer 14.17: 190.125 / 1 = 190.125

(MSA = SSA / degrees of freedom)

Question 14.18: Mean squared deviation for the columns

What is MSB, the mean squared deviation for the columns?

Answer 14.18: 6.125 / 1 = 6.125

(MSB = SSB / degrees of freedom)

Question 14.19: Mean squared deviation for the interaction

What is MSAB, the mean squared deviation for the interaction?

Answer 14.19: 3.125 / 1 = 3.125

(MSAB = SSAB / degrees of freedom)

Question 14.20: Mean squared error

What is MSE, the mean squared error?

Answer 14.20: 127.50 / 4 = 31.875

(MSE = SSE / degrees of freedom)

Question 14.21: *F* value

What is $f_A$, the *f* value for testing significance of the row differences?

Answer 14.21: 190.125 / 31.875 = 5.965

($f_A$, the *f* value for testing significance of the row differences, is MSA / MSE)

Question 14.22: *F* value

What is $f_B$, the *f* value for testing significance of the column differences?

Answer 14.22: 6.125 / 31.875 = 0.192

($f_B$, the *f* value for testing significance of the column differences, is MSB / MSE)

Question 14.23: *F* value

What is $f_{AB}$, the *f* value for testing significance of the interaction effect?

Answer 14.23: 3.125 / 31.875 = 0.098

($f_{AB}$, the *f* value for testing significance of the interaction effect, is MSAB / MSE)