MS Module 9 Difference of means for small samples practice exam questions

(The attached PDF file has better formatting.)

Samples from two groups have the following samples sizes, means, and standard deviations:

Group	*Sample Size*	*Sample Mean*	*Sample SD*
Group 1	5	12	1.1
Group 2	7	15	1.4

μ_1 = the mean of Group #1; μ_2 = the mean of Group #2.
The null hypothesis is H_0: $\mu_1 = \mu_2$; the alternative hypothesis is H_a: $\mu_1 \neq \mu_2$.

Question 9.1: Variance of estimated mean

What is the variance of the estimated mean of each group?

Answer 9.1: variance of the estimated mean = (standard deviation)^2 / number of observations

- Group 1: $1.1^2 / 5 = 0.242$
- Group 2: $1.4^2 / 7 = 0.280$

Question 9.2: Variance of estimated difference of group means

What is the variance of the estimated difference of the group means?

Answer 9.2: 0.242 + 0.280 = 0.522

(variance of the estimated difference of the group means = sum of variances of estimated group means)

Question 9.3: Standard deviation of estimated difference of group means

What is the standard deviation of the estimated difference of the group means?

Answer 9.3: $0.522^{0.5} = 0.7225$

(standard deviation = square root of the variance)

Question 9.4: Degrees of freedom

What are the degrees of freedom for a *t* test of each group's mean?

Answer 9.4: Degrees of freedom = number of observations – 1

- Group 1: 5 – 1 = 4
- Group 2: 7 – 1 = 6

Question 9.5: Degrees of freedom

What are the degrees of freedom for a *t* test of the difference of the group means?

Answer 9.5: The approximate degrees of freedom is

$(s^{21}/m + s^{22}/n)^2 / (s^{21}/m)^2/(m-1) + (s^{22}/n)^2/(n-1) =$

(variance of group 1 mean + variance of group 1 mean)^2 /
    (square of variance of group 1 mean / (group 1 observations – 1)
+   square of variance of group 2 mean / (group 2 observations – 1)) =

$$(0.242 + 0.280)^2 / (0.242^2 / (5 – 1) + 0.28^2 / (7 – 1)) = 9.834$$

We truncate the degrees of freedom to 9.

Question 9.6: *t* value for difference of group means

What is the *t* value for a 90% two-sided confidence interval of the difference of the group means?

Answer 9.6: 1.833

(the *t* value for a 90% two-sided confidence interval is the *t* value for a 5% one-tailed test with 9 degrees of freedom)

Question 9.7: Confidence interval

What is the 90% two-sided confidence interval of the difference of the group means ($\mu_1 – \mu_2$)?

Answer 9.7: Confidence interval = ($\mu_1 – \mu_2$) ± *t* value × standard deviation of the difference of the group means

- lower bound: (12 – 15) – 1.833 × 0.7225 = -4.324
- upper bound: (12 – 15) + 1.833 × 0.7225 = -1.676