MS Module 8 Difference of means practice exam questions

We test if the mean values in two groups differ. The number of observations in each group (sample size), the sample means, and the standard deviation of the sample values (sample SD) are shown below.

Group	*Sample Size*	*Sample Mean*	*Sample SD*
Group #1	118	23.99	25.74
Group #2	155	25.27	28.72

μ_1 = the mean of Group #1; μ_2 = the mean of Group #2.
The null hypothesis is H_0: $\mu_1 = \mu_2$; the alternative hypothesis is H_a: $\mu_1 \neq \mu_2$.

Question 8.1: Variance of difference of sample means

What is the variance of the difference (group #1 – group #2) of the two sample means?

Answer 8.1: $25.74^2 / 118 + 28.72^2 / 155 = 10.936348$

(variance of the difference of independent samples = sum of the variances of each sample)

**Question 8.2: Standard deviation of difference of sample means

What is the standard deviation of the difference (group #1 – group #2) of the two sample means?

Answer 8.2: $10.936348^{0.5} = 3.307015$

(standard deviation = square root of variance)

**Question 8.3: *z* value

What is the *z* value for the difference (group #1 – group #2) in the two sample means?

Answer 8.3: (23.99 – 25.27) / 3.307015 = -0.3871

(*z* value = difference in means / standard deviation of this difference)

**Question 8.4: *p* value

What is the *p* value for a two-tailed test of the null hypothesis?

Answer 8.4: $2 \times (1 - \Phi(0.3871)) = 2 \times (1 - 0.65066) = 0.6987$

Interpolating in the statistical tables:

$\Phi(0.38) = 0.6480$
$\Phi(0.39) = 0.6517$
$\Phi(0.3871) = ((0.3871 - 0.38) \times 0.6517 + (0.39 - 0.3871) \times 0.6480) / (0.39 - 0.38) = 0.6506$

**Question 8.5: Confidence interval for difference in means

What is the lower bound of the two-sided 90% confidence interval for $\mu_1 - \mu_2$ (the mean of Sample #1 minus the mean of Sample #2)?

Answer 8.5: (23.99 – 25.27) – 1.645 × 3.307015 = -6.720

(difference in sample means – critical *t* value × standard deviation of the difference)

Question 8.6: Probability of a Type II error

If we use an α of 10% to test the null hypothesis, and the true difference in the means $\mu_2 - \mu_1$ (the mean of Sample #2 minus the mean of Sample #1) = 4.32, what is β, the probability of a Type II error?

Answer 8.6: The probability of a Type II error for the difference in means =

$$\Phi(z_{\alpha/2} - (\Delta' - \Delta_0) / \sigma) - \Phi(-z_{\alpha/2} - (\Delta' - \Delta_0) / \sigma) =$$
$$\Phi(1.645 - 4.32 / 3.307015) - \Phi(-1.645 - 4.32 / 3.307015) =$$
$$\Phi(0.3387) - \Phi(-2.9513) = 0.631$$

! 1.645 – 4.32 / 3.307015 = 0.3387
! –1.645 – 4.32 / 3.307015 = -2.9513

$\Phi(0.3387) = 0.6326$

Interpolating in the statistical tables:

$\Phi(0.33) = 0.6293$
$\Phi(0.34) = 0.6331$
$\Phi(0.3387) = ((0.3387 - 0.33) \times 0.6331 + (0.34 - 0.3387) \times 0.6293) / (0.34 - 0.33) = 0.6326$

$\Phi(-2.9513) = 0.0016$

Interpolating in the statistical tables:

$\Phi(2.95) = 0.9984$
$\Phi(2.96) = 0.9985$
$\Phi(-2.9513) = 1 - ((2.9513 - 2.95) \times 0.9985 + (2.96 - 2.9513) \times 0.9984) / (2.96 - 2.95) = 0.0016$