MS Module 5 *t* values and two confidence intervals practice exam questions

A statistician estimates the population mean for a normal distribution from a sample of 6 points.

- The upper bound of the 99% confidence interval for the population mean is 5.78.
- The lower bound of the 95% confidence interval for the population mean is 1.4.

Question 5.1: Critical *t* value

What is the critical *t* value for a 99% confidence interval from a sample of 6 points?

Answer 5.1: 4.0321 (table look-up)

Question 5.2: Critical *t* value

What is the critical *t* value for a 95% confidence interval from a sample of 6 points?

Answer 5.2: 2.5706 (table look-up)

Question 5.3: Standard error of estimated mean

What is the standard error of the estimated mean of the population?

Answer 5.3: (5.78 – 1.4) / (4.0321 + 2.5706) = 0.6634

Let μ be the mean of the sample and Z be the standard error of the estimated mean of the population. Form two equations:

- From the 99% confidence interval: $5.78 - \mu = 4.0321 \times Z$
- From the 95% confidence interval: $\mu - 1.40 = 2.5706 \times Z$

Adding the two equations gives $(5.78 - 1.40) = (4.0321 + 2.5706) \times Z$, so

$Z = (5.78 - 1.40) / (4.0321 + 2.5706)$

(standard error of the estimated mean of the population = (upper bound of first confidence interval – lower bound of second confidence interval) / (critical *t* value of first confidence interval + critical *t* value of second confidence interval)

Question 5.4: Standard deviation of the sample

What is the standard deviation of the sample?

Answer 5.4: $0.6634 \times 6^{0.5} = 1.625$

(standard deviation of the sample = standard error of the estimated mean of the population × square root of the number of observations in the sample)

Question 5.5: Estimated mean of the population

What is the estimated mean of the population?

Answer 5.5: (two formulas:)

- $5.78 – 4.0321 \times 0.6634 = 3.105$
- $1.4 + 2.5706 \times 0.6634 = 3.105$

(estimated mean of the population = upper bound of the confidence interval – critical *t* value × standard error of the estimated mean OR lower bound of the confidence interval + critical *t* value × standard error of the estimated mean)