MS Module 4 pps mcq type 1 and type 2 errors practice exam questions

(The attached PDF file has better formatting.)

A population has a normal distribution with a mean μ_0 of 60 and a standard deviation of 7.3

One group from this population has been treated to reduce its mean; we assume it is still normally distributed with the same standard deviation. A sample of size 22 from this treated group has a sample mean of $\overline{x}$ and a true mean of μ'.

- The null hypothesis is H_0: $\mu' = \mu_0$.
- The one-sided alternative hypothesis is H_a: $\mu' < \mu_0$.

We reject the null hypothesis if $\overline{x} \leq 58.8$

Question 4.1: Standard deviation of sample mean

What is the standard deviation of the sample mean?

Answer 4.1: $7.3 / 22^{0.5} = 1.556$

(standard deviation of the sample mean = standard deviation / (number of observations in sample)$^{0.5}$)

Question 4.2: *z* statistic

What is the *z* statistic value to test the null hypothesis?

Answer 4.2: (58.8 – 60) / 1.556 = -0.771

(the *z* statistic value to test the null hypothesis = (sample mean – mean assumed in null hypothesis (μ_0)) / standard deviation of the sample mean)

Question 4.3: Probability of Type I error

What is the probability of a Type I error for this one-sided (lower-tailed) test?

Answer 4.3: $\Phi(-0.771) = 0.2203$

Interpolating in the statistical tables:

$\Phi(0.77) = 0.7794$
$\Phi(0.78) = 0.7823$
$\Phi(-0.771) = 1 - ((0.771 - 0.77) \times 0.7823 + (0.78 - 0.771) \times 0.7794) / (0.78 - 0.77) = 0.2203$

Question 4.4: Probability of Type II error

If the true mean of the sample μ' is 59.3 , what is the probability of a Type II error for this test?

Answer 4.4: (59.3 – 58.8) / 1.556 = 0.3213, so $\Phi(59.3 - 58.8) / 1.556 = \Phi(0.3213) = 0.6260$

Interpolating in the statistical tables:

$\Phi(0.32) = 0.6255$
$\Phi(0.33) = 0.6293$

$\Phi(0.3213) = ((0.3213 – 0.32) \times 0.6293 + (0.33 – 0.3213) \times 0.6255) / (0.33 – 0.32) = 0.6260$