MS Module 7 Hypothesis testing of proportions practice exam questions

(The attached PDF file has better formatting.)

The average proportion of death from a disease is 79% . A study tests whether a drug reduces the proportion of death from the disease. Of 100 subjects who are given the drug, 73 die from the disease. Let μ be the expected proportion of death from the disease among subjects given the drug.

! The null hypothesis is H_0: the expected proportion of subjects dying $\mu = \mu_0 = 79\%$
! The one-tailed alternative hypothesis is H_a: the expected proportion dying $\mu < \mu_0$
! The two-tailed alternative hypothesis is H_a: the expected proportion dying $\mu \neq \mu_0$

The null hypothesis is tested at a 1% significance level and the true incidence of death with the drug is 68%.

Question 7.1: Sample mean

What is the incidence of death from the disease in the sample?

Answer 7.1: 73 / 100 = 73%

Question 7.2: Standard deviation

What is the standard deviation of the incidence of death in the sample if the null hypothesis is true?

Answer 7.2: $(79\% \times (1 – 79\%) / 100)^{0.5} = 0.040731$

(variance of proportion = p(1-p)/n; use proportion assumed in null hypothesis)

Question 7.3: *z* value

What is the *z* value used to test the null hypothesis?

Answer 7.3: (73% – 79%) / 0.040731 = -1.4731

(*z* value = sample mean – μ_0 (mean assumed in the null hypothesis) / standard deviation of the sample mean)

Question 7.4: *p* value for one-tailed alternative hypothesis

What is the *p* value for the one-tailed alternative hypothesis?

Answer 7.4: $\Phi(-1.4731) = 0.0704$

Interpolating in the statistical tables:

$\Phi(1.47) = 0.9292$
$\Phi(1.48) = 0.9306$

$\Phi(–1.4731) = 1 – ((1.4731 – 1.47) \times 0.9306 + (1.48 – 1.4731) \times 0.9292) / (1.48 – 1.47) = 0.0704$

Question 7.5: *p* value for two-tailed alternative hypothesis

What is the p value for the two-tailed alternative hypothesis?

Answer 7.5: $2 \times 0.0704 = 0.1408$ (0.1407 if computations carried to more decimal places)

Question 7.6: Expected value and variance of the z value

What are the expected value and variance of the z value if the null hypothesis is true?

Answer 7.6: $\mu = 0$; $\sigma^2 = 1$

(If the null hypothesis is true, the z value has a standard normal distribution: mean = zero and variance = 1)

Question 7.7: Standard deviation of sample mean

What is the standard deviation of the sample mean if the true incidence of death with the drug is 68%?

Answer 7.7: $(68\% \times (1 – 68\%) / 100)^{0.5} = 0.046648$

Question 7.8: Expected value of the z value

What are the expected value of the z value for testing the null hypothesis if the true incidence of death with the drug is 68%?

Answer 7.8: $(68\% – 79\%) / 0.040731 = -2.7006$

Question 7.9: Standard deviation of the z value

What is the standard deviation of the z value for testing the null hypothesis if the true incidence of death with the drug is 68%?

Answer 7.9: $0.046648 / 0.040731 = 1.1453$

Question 7.10: Probability of Type II error for one-tailed test

What is the probability of a Type II error for the one-tailed test?

Answer 7.10: The probability of a Type II error when the true proportion is p' for the one-tailed test is

$$1 – \Phi[\ (p_0 – p' – z_\alpha \times (p_0\ (1 – p_0) / n)^{0.5}\ ) / (\ (p'(1 – p') / n)^{0.5}\ ]$$

$(p_0 – p' – z_\alpha \times (p_0\ (1 – p_0) / n)^{0.5}\ ) / (\ (p'(1 – p') / n)^{0.5} = (79\% – 68\% – 2.326 \times 0.040731) / 0.046648 = 0.3271$

$z_\alpha = 2.326$ (lower limit of right 1% tail)

$1 – \Phi(0.3271) = 0.3718$

Interpolating in the statistical tables:

$\Phi(0.32) = 0.6255$
$\Phi(0.33) = 0.6293$

$1 - \Phi(0.3271) = 1 - ((0.3271 - 0.32) \times 0.6293 + (0.33 - 0.3271) \times 0.6255) / (0.33 - 0.32) = 0.3718$

Question 7.11: Probability of Type II error for two-tailed test

What is the probability of a Type II error for the two-tailed test?

Answer 7.11: The probability of a Type II error when the true proportion is p' for the two-tailed test is

$$\Phi[(p_0 - p' + z_{\alpha/2} \times (p_0 (1 - p_0) / n)^{0.5}) / ((p'(1 - p') / n)^{0.5})$$

$$- \Phi[(p_0 - p' - z_{\alpha/2} \times (p_0 (1 - p_0) / n)^{0.5}) / ((p'(1 - p') / n)^{0.5})$$

$(p_0 - p' + z_{\alpha/2} \times (p_0 (1 - p_0) / n)^{0.5}) / ((p'(1 - p') / n)^{0.5} = (79\% - 68\% + 2.576 \times 0.040731) / 0.046648 = 4.6073$

$(p_0 - p' - z_{\alpha/2} \times (p_0 (1 - p_0) / n)^{0.5}) / ((p'(1 - p') / n)^{0.5} = (79\% - 68\% - 2.576 \times 0.040731) / 0.046648 = 0.1088$

$z_{\alpha/2} = 2.576$ (lower limit of right 0.5% tail)

Interpolating in the statistical tables:

$\Phi(4.6073) \approx 1.000$

$1 - \Phi(0.1088) = 0.4567$

$\Phi(0.10) = 0.5398$
$\Phi(0.11) = 0.5438$

$1 - \Phi(0.1088) = 1 - ((0.1088 - 0.10) \times 0.5438 + (0.11 - 0.1088) \times 0.5398) / (0.11 - 0.10) = 0.4567$

Question 7.12: Observations needed for one-tailed test

How many observations are needed for a one-tailed test if $\alpha = 1\%$ and $\beta = 5\%$?

Answer 7.12: The number of observations needed for a one-tailed test =

$$((z_\alpha \times (p_0 (1 - p_0))^{0.5} + z_\beta \times (p'(1 - p'))^{0.5}) / (p_0 - p'))^2$$

For $\alpha = 1\%$ and $\beta = 5\%$, this equals

$$((2.326 \times (0.79 \times (1 - 0.79))^{0.5} + 1.645 \times (0.68 \times (1 - 0.68))^{0.5}) / (0.79 - 0.68))^2 = 243.006$$

The next highest integer is 244.

Question 7.13: Observations needed for two-tailed test

How many observations are needed for a two-tailed test if $\alpha = 1\%$ and $\beta = 5\%$?

Answer 7.13: The number of observations needed for a two-tailed test =

$$((z_{\alpha/2} \times (p_0 (1 - p_0))^{0.5} + z_\beta \times (p'(1 - p'))^{0.5}) / (p_0 - p'))^2$$

For $\alpha = 1\%$ and $\beta = 5\%$, this equals

$$((2.576 \times (0.79 \times (1 - 0.79))^{0.5} + 1.645 \times (0.68 \times (1 - 0.68)\ )^{0.5}) / (0.79 - 0.68)\ )^2 = 272.724$$

The next highest integer is 273.