

## Corporate Finance, Module 3: "Common Stock Valuation"

### *Illustrative Test Questions and Practice Problems*

(The attached PDF file has better formatting.)

These problems combine common stock valuation (module 3) with related concepts, such as risk, return, and the opportunity cost of capital (module 6) and risk and return (module 7). The corporate finance modules are inter-related, and the final exam questions mix concepts from different modules.

#### Question 3.1: Common Stock Investments

Which of the following statements is correct?

- A. If there are no taxes, expenses, or other market imperfections, the present value of investing in a stock depends on how long one holds the stock.
- B. The market capitalization rate for a stock is the opportunity cost of capital of investing in the stock.
- C. Since a stock provides uncertain cash flows, the expected return on a stock is greater than the risk-free interest rate.
- D. If the expected return on Stock Y exceeds the expected return on Stock Z, the expected standard deviation of the returns on Stock Y exceeds the expected standard deviation of the returns on Stock Z.
- E. None of A, B, C, or D is true.

Answer 3.1: B

*Statement A:* If the market is efficient, the present value of investing in a stock is zero, regardless of how long the investor plans to hold the stock. If taxes or expenses are considered, the longer one holds the stock, the more valuable the investment.

- ! Taxes are paid when the capital gains are realized, so the longer one holds the stock, the lower is the effective tax rate and the greater is the return from the stock.
- ! Expenses are incurred when the stock is traded, so the longer one holds the stock, the lower are the expenses, and the greater is the return from the stock.

Since taxes and expenses vary by type of investor, Brealey and Myers discuss the non-tax, no expenses scenario. We discuss taxes in the capital structure modules. Taxes are complex, and the empirical evidence is not easy to reconcile with the theory, so Brealey and Myers can not always state definitive conclusions about the effects of taxes.

*Statement B:* Suppose the statement were not true. If the market capitalization rate were higher than the opportunity cost of capital, investors earn more from the stock than they earn from other investments of similar risk. They bid up the price of the stock until its capitalization rate declines to the opportunity cost of capital. If the market capitalization rate were lower than the opportunity cost of capital, investors earn less from the stock than they earn from other investments of similar risk. They bid down the price of the stock until its capitalization rate rises to the opportunity cost of capital.

*Statement C:* If the beta of a stock is less than zero, the expected return on the stock is less than the risk-free interest rate, despite the volatility of the stock returns. The statement should say: "If the correlation between a stock's return and the overall market return is positive, the expected return on a stock is greater than the risk-free interest rate."

*Question:* Do any stocks have negative betas?

*Answer:* Some analysts believe that gold stocks may have negative betas; others disagree. No other stocks seem to have negative betas.

*Question:* How do we know that only positive beta stocks have expected returns above the risk-free rate? Perhaps all stocks have expected returns above the risk-free rate.

*Answer:* Brealey and Myers are strong advocates of the CAPM, which implies that only positive beta stocks have expected returns above the risk-free rate. We don't have empirical evidence proving this.

*Statement D:* The expected return varies with the systematic risk of a stock, not with its standard deviation. Consider two stocks, Y and Z, with the following traits:

!  $\sigma_y = 40\%$  and  $(r_y, r_m) = 10\% \Rightarrow \text{covariance}(r_y, r_m) = 40\% \times 10\% / \sigma_m^2 = 4\% / \sigma_m^2$   
 !  $\sigma_z = 20\%$  and  $(r_z, r_m) = 40\% \Rightarrow \text{covariance}(r_z, r_m) = 20\% \times 40\% / \sigma_m^2 = 8\% / \sigma_m^2$

The standard deviation is twice as high for stock Y as for stock Z, but the  $\beta$  for stock Z is twice as high as the  $\beta$  for stock Y.

*Question:* What must we know about variances, covariances, and correlations?

*Answer:* Know two relations:

$$\text{covariance}(r_y, r_m) = (r_y, r_m) \times \sigma_y \times \sigma_m \quad \text{and} \quad \beta_y = \text{covariance}(r_y, r_m) / \sigma_m^2$$

Combining these two equations gives us a third:  $\beta_y = (r_y, r_m) \times \sigma_y / \sigma_m$

*Question:* Must we know the derivation of these equations?

*Answer:* The first equation is the definition of the correlation. The second equation says that the CAPM beta is the slope parameter of the regression of the stock return on the market return. We cover the definition of the slope parameter in the mathematical statistics on-line course. You don't have to know the derivation of this equation for the corporate finance on-line course.

### Exercise 3.2: Dividend Growth Model

A stock has a capitalization rate of 15% per annum. Next year's dividend is \$10, and the dividends are expected to grow indefinitely at 5% per annum. What is the value of the stock?

Solution 3.2: 
$$P_0 = \text{Div}_1 / (k - g) = \$10 / (15\% - 5\%) = \$100.$$

*Question:* If the problem had said that this year's dividend is \$10, what would be the value of the stock?

*Answer:* If this year's dividend is \$10, next year's dividend is  $\$10 \times 1.05 = \$10.50$  and the value of the stock is  $\$100 \times 1.05 = \$105$ .

*Question:* As the capitalization rate or the dividend growth rate change, what happens to the stock price?

*Answer:* As the capitalization rate increases, the stock price decreases; as the dividend growth rate increases, the stock price increases.

*Question:* What happens if the dividend growth rate equals the capitalization rate?

*Answer:* Suppose the risk-free rate is 8% per annum and a perpetual Treasury bond pays coupons that increase 8% a year. The value of this bond is infinite. If Treasury bond trade in a competitive market, their value cannot be infinite, so the growth rate can not be same as the risk-free rate.

*Question:* What prevents this from happening?

*Answer:* Suppose the risk-free rate is 8% per annum and the government offers a bond that pays a coupon increasing 8% a year. The value of the bond is infinite, so no matter what the price of the bond is, more investors want to buy the bonds than the government has to sell. The investors bid down the yield on the bond: some investors offer to pay the asking price for a lower coupon yield, such as a 7% or a 6% increasing coupon. In an efficient market, enough investors will offer to buy the bond for a coupon yield that correctly reflects its true worth.

### Question 3.3: Dividend Growth Model

Which of the following is the most likely use of the dividend growth model?

- A. To determine the value of a stock
- B. To determine the capitalization rate for a stock
- C. To determine the growth rate of the stock's dividends
- D. To determine the return on book equity
- E. To determine the price-earnings ratio

Answer 3.3: B

*Statement A:* If the market is efficient, the price of the stock is the best indication of its worth. In theory, the dividend growth model might be used to determine if a stock is over-valued or under-valued by the market, but the uncertainty of the dividend growth rate and the market capitalization rate make this use impractical in most scenarios.

*Question:* What if an investor studies the capitalization rates and dividend yields of common stock and finds that the true value of the stock is more than its market price. Should the investor buy the stock, since its price will rise as other investors realize its true market value as well?

*Answer:* The market is efficient. Many investors study the capitalization rates and dividend yields of common stock, and each hopes to discern the true value. The market price is the average of all investors' beliefs. If 1,000 investors believe the true market value is higher than the price, 1,000 believe that it is lower. Brealey and Myers says: "Trust the market; unless you know more than everyone else, you are probably wrong."

*Question:* U.S. News and World Report studies the performance of mutual funds. Some mutual funds have out-performed the market for several years in a row. A few mutual funds have out-performed the market for the past six or seven years straight. Shouldn't we assume that these mutual funds have better managers and will out-perform the market in the coming years as well?

*Answer:* In any year, half the funds will out-perform the market and half will do worse. If 1,024 funds compete, we expect that 512 will out-perform the market any year, 256 fund will out-perform the market two years in a row, ..., and 1 will out-perform the market 10 years in a row. This is chance, not manager abilities. Given these results, we assume all the funds are equal, and their good or bad performance is chance.

*Question:* What would indicate that mutual fund manager differ in their abilities to pick stocks?

*Answer:* If 100 of the funds have better performance 10 years in a row, we assume this is not chance, but reflects different abilities. But the empirical evidence suggests that chance is the explanation for differing performance, not managerial ability.

*Question:* It seems that the number of funds that out-perform the market is higher than chance along would have us expect; why is that?

*Answer:* Firms that seem to do well continue; firms that do poorly change their names. Suppose that 1,024 firms start in 20X0. If a firm does better than average, it continues the next year. Of firms that do worse than average, only half continue the next year.

If firms do better or worse than average purely by chance, after 8 years, 4 firms have done better than average in all 8 years. But only a few firms have 8 years of experience. Of the four firms that did better than average the first 7 years and worse than average the eighth year, 2 have closed and two still continue. Only a few firms have 8 years of experience and still continue. It seems like the percentage that out-perform the market for 8 years running is too high, but it may be just right.

## *ACTUAL PERFORMANCE*

*Statement B:* The capitalization rate for the stock is hard to estimate; we use the dividend growth model to estimate the capitalization rate.

*Question:* Why not use the CAPM formula, which says that the capitalization rate depends on the beta, which is the covariance of the stock's return with the market return divided by the variance of the market return? This relieves us of having to forecast future dividend streams.

*Answer:* There are several reasons:

The covariance of the stock's return with the market return is stochastic, affected by chance fluctuations in stock returns. The random fluctuations are so high that these estimates of beta are highly uncertain.

The covariance of the stock's returns with the market returns may be distorted by random fluctuations and by changes in the beta of the stock over time. We might use the dividend growth rate to estimate the market capitalization rate for the stock.

*Statement C:* Few stocks expect a constant dividend growth rate indefinitely. Unless we know the pattern of dividend payments, we can't use the dividend growth model to estimate the growth rate.

*Statement D:* The return on book equity is an accounting ratio. It can be read off the firm's income statement; we do not estimate it from uncertain inputs like the dividend growth rate and the capitalization rate of the stock.

*Statement E:* The price-earnings ratio is a market value that is determined from the firm's income statement along with its share price. We do not estimate the price-earnings ratio from uncertain inputs like the dividend growth rate and the capitalization rate of the stock.

*Question:* Why might actuaries use the dividend growth model?

*Answer:* The dividend growth model is often used in personal auto rate hearings to select a capitalization rate for the insurance company.

### Exercise 3.4: Dividend Growth Model

A stock stands now at  $P_0 = \$73$ . Analysts expect earnings and dividends to grow indefinitely at 8.5% per annum, and the firm *now* pays dividends ( $D_0$ ) of \$1.68 per share.

- A. What is the expected dividend *next year*?
- B. What is the expected capitalization rate for this stock?
- C. If the risk-free rate is 5% and the expected return on the market portfolio is 12%, what is the beta of this stock?

Solution 3.4:

*Part A:* This year's dividend is \$1.68 per share, so next year's expected dividend is  $\$1.68 \times 1.085 = \$1.82$ .

*Part B:* The capitalization rate is

$$\$1.82 / \$73 + 8.5\% = 10.99\% \approx 11\%.$$

*Part C:* The CAPM gives

$$11\% = 5\% + \beta \times (12\% - 5\%) \Rightarrow \beta = (11\% - 5\%) / (12\% - 5\%) = 0.857.$$

### Exercise 3.5: Dividend Growth Model

Suppose we have the following values for a stock:

!	$P_0$	= \$80
!	$Div_1$	= \$10
!	Dividend growth rate	= 4% per annum
!	$\sigma_{2m}$	= 40%
!	Covariance( $r_s, r_m$ )	= 30%
!	Return on book equity	= 12%
!	Payout ratio	= 66.7%
!	Risk-free interest rate	= 5%
!	Market risk premium	= 7%

Which of the following is the most likely inference from these figures?

- A. The dividend growth rate is really more than 4%.
- B. The variance of the market returns is really less than 40%.
- C. The covariance of the stock's returns with the market returns is really more than 30%.
- D. The return on book equity is really more than 12%.
- E. The payout ratio is really less than 66.7%.

Answer 3.5: C

The dividend growth rate ( $g$ ), return on book equity (ROE), and payout ratio ( $p$ ) are linked in that  $g = ROE \times (1 - p)$ , as is true in the exercise:  $4\% = 12\% \times (1 - 66.7\%)$ . We have no reason to assume that any of these figures is overstated or understated.

- ! The variance, covariance, and beta are linked in that  $\beta = Cov(r_s, r_m) / Var(r_m)$ . From the figures in the exercise,  $\beta = 30\% / 40\% = 0.750$ .
- ! The beta is linked to the capitalization rate ( $k$ ) as  $k = r_f + \beta \times (E[r_m] - r_f) = 5\% + \beta \times 7\%$ .
- ! The dividend growth model gives the capitalization rate as the dividend yield plus the growth rate =  $\$10 / \$80 + 4\% = 16.5\%$ .
- ! A beta of 0.750 gives a market capitalization rate of  $5\% + 0.750 \times 7\% = 10.25\%$ .

These figures do not agree with each other. If the beta were larger, the CAPM would give a higher market capitalization rate. The beta would be larger for either of two reasons:

- ! The covariance of the stock return and the market return is larger.
- ! The variance of the market return is smaller.

The covariance is most uncertain figure in the list, and it is easily distorted by random fluctuations and by mean reversion. (The dividend growth rate is also uncertain, but in this scenario it is supported by the return on book equity, which is reasonably in line with the overall market return, and the payout ratio, which is reasonable for many stocks.)

### Exercise 3.6: Dividend Growth Model

A stock is standing at \$73, and the firm is now paying dividends of \$1.68 per share. The firm earns 12% on book equity, and it pays out 50% of earnings as dividends. The return on equity and the payout ratio are expected to continue indefinitely.

- A. What is the expected growth rate of earnings and dividends?
- B. What is the expected capitalization rate for this stock?
- C. If the risk-free rate is 5% and the expected return on the market portfolio is 12%, what is the beta of this stock?

Solution 3.6:

Part A: The expected growth rate is  $12\% \times 50\% = 6\%$ .

Part B: This year's dividend is \$1.68 per share, so next year's expected dividend is  $\$1.68 \times 1.06 = \$1.78$ . The capitalization rate is  $\$1.78 / \$73 + 6\% = 8.44\%$ .

Part C: The CAPM gives  $8.44\% = 5\% + \beta \times (12\% - 5\%) \Rightarrow \beta = (8.44\% - 5\%) / (12\% - 5\%) = 0.491 \approx 50\%$ .



### Exercise 3.7: Dividend Growth Model

Next year, a stock will pay a dividend of \$2 per share, after which dividends are expected to grow by 4% per annum (in perpetuity). If an investor requires a return of 12% per annum to invest in this stock, how much should he be willing to pay per share?

Solution 3.7:

Using the dividend growth model,  $P_0 = \text{Div}_1 / (k - g) = \$2 / (12\% - 4\%) = \$25$ .

### Exercise 3.8: Relative Values

- ! Stock A is expected to provide a dividend of \$10 a share forever.
  - ! Stock B is expected to pay a dividend of \$5 next year, after which dividends should grow by 4% per annum indefinitely.
  - ! Stock C is expected to pay a dividend of \$5 next year, after which dividends should grow by 20% per annum for five years (i.e., until year 6) and then remain level.
- A. If the market capitalization rate for each stock is 10%, which stock is the most valuable?  
B. If the market capitalization rate for each stock is 7%, which stock is the most valuable?  
C. At what capitalization rates is Stock A the most valuable?

Solution 3.8:

Part A: At a 10% market capitalization rate:

For stock C, let  $v = 1/1.10$  and  $g = 1.20$ , so  $gv = 1.20 / 1.10 = 109.09\% \approx 1.09$ .

- !  $P_A = \$10 / 0.10 = \$100$
- !  $P_B = \$5 / (0.10 - 0.04) = \$83.33$
- !  $P_C = \$5 \times v \times (1 + 1.0909 + 1.0909^2 + 1.0909^3 + 1.0909^4 + 1.0909^5 / 0.10) = \$104.50$

Stock C is the most valuable.

Part B: At a 7% market capitalization rate:

For stock C, let  $v = 1/1.07$  and  $g = 1.20$ , so  $gv = 1.20 / 1.07 = 1.1215$ .

- !  $P_A = \$10 / 0.07 = \$142.86$
- !  $P_B = \$5 / (0.07 - 0.04) = \$166.67$
- !  $P_C = \$5 \times v \times (1 + 1.1215 + 1.1215^2 + 1.1215^3 + 1.1215^4 + 1.1215^5 / 0.07) = \$156.50$

Stock B is the most valuable.

Part C: This computation is not tested on the final exam; we explain the method here.

Comparing Stocks A and B is easy. For a high capitalization rate, Stock A is worth more; for a low capitalization rate, Stock B is worth more. (If the capitalization rate is 4%, the value of Stock B is unbounded. Let  $k =$  the capitalization rate. Solve for the capitalization rate at which the two stocks have the same value as  $\$10 / k = \$5 / (k - 4\%)$ ).

Comparing stocks A and C is more mathematics but the same idea. For stock C, let  $v = 1/(1 + k)$  and  $g = 1.20$ , so  $gv = 1.20 / (1 + k)$ . Use the Excel *SOLVER* add-in to find  $k$ . This is not a pencil and paper problem.

The final exam might compare two simple stocks (like A and B); it won't compare more complex dividend streams, as in Stock C.