Corporate Finance, Module 6: "Risk, Return, and Cost of Capital"

Homework Assignment

(The attached PDF file has better formatting.)

An investor has a three stock portfolio.

Stock	Percentage of Portfolio	Expected Return	Standard Deviation	Correlations among Stocks		
				Stock A	Stock B	Stock C
A	50%	10%	20%	1.0	0.5	0.3
B	30%	15%	30%	0.5	1.0	0.1
C	20%	20%	40%	0.3	0.1	1.0

A. What is the expected return for the three stock portfolio? (The expected return is the weighted average of the expected returns of each stock.)
B. What is the variance of the return of the three stock portfolio? (We determine the variance of the three stock portfolio as the variance of a combination of three random variables. When computing the variance, make sure to use the square of the percentage held as the weights for the variance terms, and the product of the percentages held as the weights for the covariance terms. The correlations are used in the covariance terms. See the practice problems for this module for the formulas.)
C. What is the standard deviation of the return of the three stock portfolio? (The standard deviation is the square root of the variance.)

Question: How do we determine the correlation between the returns on two stocks? In this homework assignment, we are given the correlations. In practice, how do we derive them?

Answer: The correlation $\rho_{a,b}$ is defined as the covariance between the returns on stocks A and B divided by the product of the standard deviations of stocks A and B:

$$\rho_{a,b} = \text{covariance}(A,B) / (\sigma_a \times \sigma_b)$$

If we have a sample of returns, such as returns for the 252 trading days in the year, the correlation is

$$\rho_{a,b} = \frac{\sum r_a r_b}{\sqrt{\sum r_a^2 \sum r_b^2}}$$

Question: Do we use this formula to derive correlations between stocks?

Answer: The high stochasticity of stock returns causes random fluctuations to overwhelm the true correlation. Many analysts assume the same correlation exists between any two firms in an industry. We examine the correlations of several large stock insurers, take the average, and assume the average holds for all insurers.

Question: The correlation between two auto insurers is greater than that between an auto insurer and a life insurer. Averages from all large insurers may not be correct.

Answer: We use averages of insurers of the same type for the expected correlation among insurers of the same type and averages of insurers of different types for the expected correlation among insurers of different types. But you are correct: every pair of insurers is has its own correlation, and averages are not always a good proxy. Much of the correlation estimate is judgment; we don't always have good data.

(This problem is not required homework. But you will use the concept of systematic risk many times in your actuarial career, and you will be asked to explain why only systematic risk warrants additional return.)

The distinction between systematic risk and unique risk affects the proper capitalization rate.

Question: The notion that only systematic risk warrants additional return is puzzling. Science advances by testing hypotheses. If we think that only systematic risk warrants additional return, we form experiments and test this hypothesis. Financial analysts have tried to test this hypothesis, and they get mixed results.

!   They can't agree on the market return: the stock market, the market of traded assets, a global market, the market of all financial assets plus coins, stamps, art, precious metals, and real estate.
!   Empirical tests of the CAPM are not encouraging. Given the regard for the CAPM in the textbook, we expect the empirical tests to support the theory. Some tests support it partly, and some tests give it only lukewarm support.
!   Some tests do not support the CAPM. High beta stocks have actual returns below the CAPM indication, and low beta stocks have actual returns that are higher.

Despite all this, everyone seems to agree that only systematic risk warrants additional return. Where is the evidence for this?

Answer: This notion is supported by an arbitrage argument. The logic is compelling and few people argue with it. If the market return were proportional to total risk (both systematic risk and unique risk), arbitragers would make risk free profits.

Suppose the expected return on a security were the risk-free rate plus some constant times the *total* standard deviation of the return. An arbitrager would buy N different stocks with high unique risk, package them into a mutual fund, and sell the fund to other investors.

A.   Explain why the arbitrager earns risk-free profits. (The arbitrager buys stock with high risk, so the return is high. This means that the stock price is low. The arbitrager sells a mutual fund with low risk, so the return is low. This means that the market value of the mutual fund is high. The arbitrager earns the risk-free profits now, not later.)
B.   Explain how supply and demand eliminates the risk-free profit and eliminates the return on diversifiable risk. (Demand by arbitragers is high for the stocks with high total standard deviation. This drives up the price, reducing the return to arbitragers.)

Explain the logic in your own words. This homework assignment ensures that you understand the theory.