Corporate Finance, Module 6, “Risk, Return, and the Opportunity Cost of Capital”

Geometric Average vs Arithmetic Average

(The attached PDF file has better formatting.)

Illustration: Geometric vs Arithmetic Average

An investment returns either 0% or 20% each year, with a 50% probability of each. What is the expected annual return?

Question: Consider the expected return over two years. The investment may earn 0% the first year and 20% the next year, or 20% the first year and 0% the next year. Either way, the two year return is 20%, which is equivalent to $1.20^{½} - 1 = 9.54\%$ per annum. We use the geometric average, not the arithmetic average.

Answer: You are correct that 20% over two years is equivalent to 9.54% over one year. But the expected return over two years is not 20%. The investment has four possible returns, not two:

	Returns			
Scenario	*Year 1*	*Year 2*	*Two Year*	*Probability*
1	0%	0%	0%	25%
2	0%	20%	20%	25%
3	20%	0%	20%	25%
4	20%	20%	44%	25%
Total			84%	100%

The expected return is 84% / 4 = 21%. The expected annual return is $1.21^{½} - 1 = 10\%$. This is the arithmetic average of 0% and 20%.