

Corporate finance module 16: Debt policy (capital structure)

Brealey and Myers, *Corporate Finance*, chapter 17

(The attached PDF file has better formatting.)

Capital structure affects the returns on capital. In perfect capital markets, it affects the return on debt capital and the return on equity capital. If the corporate tax rate is positive or costs of bankruptcy are positive, it also affects the return on assets (= the weighted average cost of capital).

Final exam problems relate several items:

- ! The opportunity cost of capital = the cost of capital with all equity financing.
- ! The debt-to-equity ratio (or debt-to-value ratio or equity-to-value ratio).
- ! The cost of debt capital (or the beta of debt) at the given debt-to-equity ratio.
- ! The cost of equity capital (or the beta of equity) at the given debt-to-equity ratio.
- ! The corporate tax rate (or marginal tax rate).

The exam problem may relate returns on capital or betas of capital. The formulas are the same.

- ! The exam problem may give the risk-free rate and the market risk premium and mix returns with betas.
  - " The CAPM equation for stocks applies to debt and assets as well.
- ! The exam problem may give the share price and the number of shares; the product is equity capital.
- ! The firm may pay a one time dividend or repurchase shares.
  - " Both refinancing methods reduce equity.
  - " The one-time dividend also reduces the share price.

[Note: Final exam problems may assume perfect capital markets or imperfect capital markets. Master the procedures for perfect capital markets first; they are clearer, and are the starting point for all capital markets. If the corporate tax rate is non-zero, include the complement of the tax rate as part of the bond return in the WACC formula.]

\*\* Exercise 16.1: Capital structure

- ! A firm with all-equity financing has a beta of 1.
- ! The firm generates a level, perpetual stream of earnings and dividends.
- ! The stock has a price earnings ratio of 8 and a cost of equity of 12.5%.
- ! Shares trade at \$50.
  
- ! The firm decides to repurchase half its shares and substitute an equal value of debt.
- ! The debt is risk-free, with a 5% interest rate.
- ! The marginal tax rate  $T_c = 0$ , and there are no imperfections in capital markets

Calculate the following after the re-financing.

- A. The overall cost of capital (WACC).
- B. The stock price.
- C. The cost of equity.
- D. The price-earnings ratio
- E. The stock's beta.

*Part A:* This exercise assumes perfect capital markets, so capital structure does not affect the worth of the firm. The overall cost of capital (WACC) does not change. It was 12.5% before the refinancing, so it is 12.5% after the refinancing.

*Part B:* The worth of the firm does not change, so the stock price does not change. It was \$50 a share before the refinancing, so it is \$50 a share after the refinancing.

*Question:* The firm has bought back half its stock, so its value to shareholders should decrease by 50%. A past module in this course uses this relation: if a firm pays \$1 to shareholders (by a dividend or by a share repurchase), its value declines \$1. If the firm's share price is \$50 before a dividend and the firm pays a \$1 stockholder dividend, the share price is \$49 after the dividend.

*Answer:* Yes, the firm's value to shareholders declines 50%. But shareholders own only half as many shares, so the stock price remains \$50. The bondholders own the other half of the firm.

*Take heed:* If the the firm pays a one-time dividend to shareholders from the bond proceeds, the number of shares does not change but the value of each share declines. If the firm buys back shares, the value per share does not change but the number of shares held by investors declines.

*Part C:* The WACC is a weighted average of the cost of debt capital and the cost of equity capital. The weights in the exercise are 50% debt and 50% equity, and the cost of debt capital is 5% per annum, so

$$50\% \times 5\% + 50\% \times r_E \text{ (cost of equity capital)} = 12.5\% \\ \Rightarrow r_E \text{ (cost of equity capital)} = 20.0\%.$$

*Question:* if the marginal tax rate is more than zero, how do we work this problem?

*Answer:* First assume the marginal tax rate is zero, giving a WACC of 12.5% and a cost of equity capital of 20%. Then use the actual marginal tax rate of 35%, giving a revised WACC of

$$50\% \times 5\% \times (1 - 35\%) + 50\% \times 20\% = 11.625\%$$

*Take heed:* Many final exam problems ask for the WACC after refinancing in **imperfect** capital markets.

*Part D:* Earnings are level and perpetual, so earnings per share equal the cost of equity capital, or 20%. To show the reasoning, we solve the problem by first principles as well.

*Method:* Determine the annual earnings before refinancing, subtract the debt costs, and divide by the new equity value.

Step #1: Before the refinancing, the stock price is \$50 and the price-earnings ratio is 8, the annual earnings per share are  $\$50 / 8 = \$6.25$ .

Step #2: After refinancing, the firm has \$50 of debt for each \$50 of equity (= one share). Suppose the firm originally had two million shares, for total value of \$100 million and earnings of \$12.5 million. It now has one million shares worth \$50 million and \$50 million of debt. Interest is  $5\% \times \$50 \text{ million} = \$2.5 \text{ million}$ . Earnings are  $\$12.5 \text{ million} - \$2.5 \text{ million} = \$10 \text{ million}$ . The shares are worth \$50 million, so the price-earnings ratio is  $\$50 / \$10 = 5$ .

*Part E:* The risk-free rate is 5%. With a beta of 1, the cost of equity is 12.5%, so the market risk premium is 7.5%. If the new cost of equity capital is 20%, then  $20\% = 5\% + \beta \times 7.5\% \Rightarrow \beta = 2$ .

*Question:* The firm's value has not changed, so why does the beta change? The beta reflects the change in the firm's value for a 1% change in the overall market. The firm's assets have not changed, so its operating income does not change.

*Answer:* The beta of the firm's assets does not change, but the beta of the firm's *equity* changes. Before the refinancing, if the overall stock market increases 1%, the firm's value increases 1%. This is still true after the refinancing, since the firm's assets do not change. The bondholders' 50% portion of the firm does not change; they still get a 5% return on their investment. To compensate, the stockholders' 50% portion of the firm increases 2%.

## CAPITAL STRUCTURE: DEBT VS EQUITY

When should firms issue debt? The exercise above seems to provide an answer. One might reason:

- ! Suppose a firm is profitable and expects to earn more than the average firm in the industry.
- ! The stock trades at \$100 per share, and investors do not think this firm is more profitable than average.
- ! Other firms in the same industry earn \$10 per annum for every share of stock.
- ! This firm expects to earn \$20 per annum for each share of stock.
  
- ! The firm should say: Let us re-finance the firm with 50% debt at 5% per annum.
- ! The firm now expects to earn  $2 \times \$20 - \$5 = \$35$  per share of stock.

⇒ Profitable firms should have high debt ratios; unprofitable firms should have low debt ratios. More precisely, if the firm thinks it is better than the market expects, it should issue more debt. But this logic is specious.

In perfect capital markets, profitable firms have higher stock prices, so they have no reason to issue more debt. That is the Miller and Modigliani theorem. The reasoning above assumes the firm's managers know it is more profitable than the market expects. This is an information imperfection. Sometimes managers do have more knowledge of the firm's expected profitability than investors have; more often, managers think their firms are better than average even if they are not.

Brealey and Myers have a more subtle argument. They say that profitable firms can use the debt tax shields, so they should issue more debt. But the opposite seems true. Profitable firms have low debt ratios. Brealey and Myers explains that managers want to reduce debt ratios to have more financial slack.

*Question:* If debt raises the value of the firm, why do managers want low debt ratios?

*Answer:* Debt raises the value of the firm to stockholders. Managers have conflicting incentives.

- ! If the firm is worth more, it can invest more and grow more rapidly, increasing the value to managers. But this added value is limited. Many managers won't see any increase in their salaries.
- ! If the firm had higher debt, it has higher risk.

**\*\* Exercise 16.2: Miller and Modigliani propositions**

- A. What do the Miller and Modigliani propositions assume?
- B. How does corporate borrowing affect earnings per share and the price-earnings ratio?
- C. If the cost of debt capital  $r_D$  does not vary with the amount of debt, how does the cost of equity capital  $r_E$  vary with the amount of debt?
- D. In practice, how does the amount of debt affect the cost of debt capital  $r_D$ ?
- E. If the probability of bankruptcy is nil, does borrowing increase the cost of equity capital  $r_E$ ?

*Part A:* The Miller and Modigliani propositions assume perfect financial markets. These assumptions are that the marginal tax rate is zero, the costs of bankruptcy are zero, agency costs are zero, the firm receives no subsidies from suppliers or the government

*Part B:* The price-earnings ratio is the reciprocal of earnings per share, which is the accounting equivalent of the return on equity. Debt increases earnings per share and reduces the price-earnings ratio even in perfect capital markets. If the corporate tax rate is more than zero, the tax shields from debt make this effect stronger.

*Part C:* Miller and Modigliani's proposition 2 says that the cost of equity increases with borrowing and that the increase is proportional to  $D/E$ , the debt-to-equity ratio.

- ! In perfect capital markets,  $D/V \times r_D + E/V \times r_E = r_A$ .
- ! If the cost of debt capital does not depend on the amount of debt,  $D/V \times r_D + E/V \times r_E = k$  (a constant).
- ! This relation is true for all markets, though  $r_A$  is lower because of the debt tax shields.

Express  $r_E$  as a function of  $D$ :

$$\begin{aligned} D/V \times r_D + E/V \times r_E &= k \\ r_E &= (k - D/V \times r_D) / E/V = k \times V/E - D/E \times r_D \\ V &= E + D, \text{ so} \\ r_E &= k + D/E \times (k - r_D) \end{aligned}$$

We assumed that  $r_D$  is constant, so

$$\partial(r_E) = \partial(k) + \partial(D/E) \times (k - r_D).$$

$\partial(k)$  is zero, so the change in  $r_E$  is proportional to the change in  $D/E$ .

*Part D:* Increased borrowing raises the cost of debt capital  $r_D$ . Higher debt ratios increase the likelihood of bankruptcy, decreasing the likelihood that bondholders will receive the promised payments. To offset the lower expected payments, bondholders demand higher returns.

*Part E:* Increased borrowing leverages the firm and adds variability to earnings, even if the probability of bankruptcy is nil. The added variability is often systematic risk, since low earnings for one firm are correlated (in part) with low earnings for all firms. The higher systematic risk raises the cost of equity capital  $r_E$ .

[Many final exam problems test how refinancing to a different capital structure (ratio of debt to equity) affects  $r_D$ ,  $r_E$ , and  $r_A$ : the cost of debt capital, cost of equity capital, and the return on assets, both in perfect capital markets and in capital markets with a non-zero corporate tax rate. Final exam problems on the weighted average cost of capital and the adjusted present value often require you to derive the cost of equity capital.]

**\*\* Exercise 16.3: Corporate borrowing and rates of return**

Corporate borrowing (debt) may affect  $r_D$ ,  $r_E$ , and  $r_A$ , the cost of debt capital, cost of equity capital, and the return on assets. The effects may depend on whether capital markets are perfect. Answer each of the parts below for two assumptions: (i) the corporate tax rate is zero and capital markets are perfect, and (ii) the corporate tax rate  $>$  zero.

What is the effect of a higher debt-to-equity ratio on

- A.  $r_D$  (the cost of debt capital)
- B.  $r_A$  (the return on assets)
- C.  $r_E$  (the cost of equity capital)
- D. Earnings per share
- E. The price-earnings ratio

*Part A:* The cost of debt capital  $r_D$  increases with the ratio of debt to equity. Bondholders lose if the firm goes bankrupt, so a higher chance of bankruptcy leads to a higher cost of debt capital. Bankruptcy means the firm is unable to pay its debts, so a firm that has no debt can not go bankrupt.

*Illustration:* A firm has \$100 million of assets (plants, equipment, property). It expects to earn \$100 million next year or lose \$50 million, with equal probabilities. Its expected earnings are \$25 million, for a 25% return on assets. If the firm's opportunity cost of capital is 20%, this is a positive NPV project. If the firm is all-equity financed, it earns a 25% return on equity.

Suppose the firm has 90% debt and 10% equity (\$90 million debt and \$10 million equity) and the risk-free interest rate is 15% per annum. We work out the required cost of debt capital.

- ! If the firm earns \$100 million, the bondholders get their principal with the coupon: \$90 million  $\times$  (1 +  $r_D$ ).
- ! If the firm loses \$50 million, it goes bankrupt, since it has no cash to pay the coupon payment. The firm's remaining assets are sold for \$100 million – \$50 million and bondholders get the remaining \$50 million.

The expected return to the bondholders is  $\frac{1}{2} \times (\$90 \text{ million} \times (1 + r_D) + \$50 \text{ million}) = \$90 \text{ million} \times (1 + r_f) \Rightarrow$

$$\begin{aligned} \$90 \text{ million} \times (1 + r_D) &= 2 \times \$90 \text{ million} \times (1 + r_f) - \$50 \text{ million} \\ r_D &= [ 2 \times \$90 \text{ million} \times (1 + r_f) - \$50 \text{ million} ] / \$90 \text{ million} - 1 = 74.44\% \end{aligned}$$

This computation is not complete, since it does not consider the coupon (debt) liability, which is great here. But the point is clear: if the probability of insolvency is high, the cost of debt capital is high.

*Part B:* The return on assets is a weighted average of the return on debt capital and the return on equity capital, where the weights are the percentages of debt and equity in the firm's capital structure. In perfect capital markets, the return on assets depends on the assets, not on the capital structure. The return on assets does not change with refinancing in perfect capital markets. If the corporate tax rate is more than zero, we work out the return on assets from the cost of debt capital and the cost of equity capital; see below.

*Part C:* The cost of equity capital  $r_E$  is generally greater than the cost of debt capital  $r_D$ , for two reasons.

- ! Debt has precedence in bankruptcy. If the firm goes bankrupt, the bondholders must be paid in full before the shareholders receive anything. In many cases, the bondholders lose little, and the shareholders lose their entire investment.

! The return to the bondholders is not much correlated with market returns, so its CAPM beta is close to zero. The return to shareholders is positively correlated with market returns, so their systematic risk is positive and the CAPM beta of equity is positive.

The cost of equity capital  $r_E$  is less than the cost of debt capital  $r_D$  only if the beta of equity is so negative (because the stock is a hedge against market fluctuations) that it outweighs the bondholders' advantage in bankruptcy. This does not occur in practice. The textbook assumes  $r_E > r_D$ .

For most amounts of debt, the likelihood of bankruptcy is not material, so the cost of debt capital does not rise that much. We have

$$r_A \times (D + E) = r_D \times D + r_E \times E = \text{constant}$$
$$r_A = r_D \times D/V + r_E \times E/V = \text{constant, where } V = D + E$$

- ! If we treat the cost of debt capital as fixed, then as  $D/V$  rises ( $E/V$  declines),  $r_D < r_E$  and both  $r_A$  and  $r_D$  are constant,  $r_E$  must rise.
- ! Even if  $r_D$  is not constant, as long as it changes only slightly as  $D/V$  changes, the result still holds.

*Part D:* In perfect capital markets, if the firm issues debt and *repurchases* shares, fewer shares are traded but the share price does not change. We compute the change in the earnings per share.

To simplify, we compare two capital structures: all-equity financing (no debt) vs an equal weighting of debt and equity. Assume the share price is \$100, so if the return on equity  $r_E = R\%$  with all-equity financing, the earnings per share are \$R. We assume (as before) that  $r_D < r_E$ . With 50% debt, the firm has only half as many shares.

- ! The earnings per share *before* interest payments on the debt are  $2 \times \$R = \$100 \times 2 \times r_E$ .
- ! The earnings per share *after* interest payments on the debt are  $2 \times \$R = \$100 \times (2 \times r_E - r_D)$ .

Since  $r_E > r_D$ , earnings per share have increased.

This reasoning does *not* depend on the 50% - 50% relation of debt and equity. As an exercise, show that earnings per share increases with the debt-to-equity ratio. A final exam problem may give the earnings per share with all equity financing, the cost of debt capital, and the debt-to-equity ratio after refinancing. You work out the earnings per share after refinancing.

*Part E:* The price-earnings ratio is the reciprocal of the earnings per share. If the earnings per share increases, the price-earnings ratio decreases.

*Question:* What changes if the corporate tax rate is more than zero?

*Answer:* Complete Parts A and C the same as for perfect capital markets. The new return on assets is lower, since the weighted average cost of capital multiplies the cost of debt capital by  $(1 - T_c)$ .

For *Part D*, the earnings per share with all equity financing is an after-tax figure. Convert this to a pre-tax figure, compute the pre-tax earnings per share after refinancing, subtract the debt payments, and apply the tax rate to get the after-tax figure. The computations for earnings per share with a corporate tax rate more than zero are not reviewed in the textbook and are not asked on the final exam.

*Question:* You say the return on assets *decreases*. But the textbook says the tax shields *increase* the value of assets.

*Answer:* The non-tax cash flows stay the same. Since the return on assets decreases but the cash flows stay the same, it must be that the value of assets increases.

**\*\* Exercise 16.4: Assets, liability, and equity**

[Some basic accounting knowledge is needed for corporate finance. Know what items are assets, liabilities, and equity.]

Identify each of the following as assets, liability, and equity.

- A. Common stock owned by the firm.
- B. Corporate bonds owned by the firm.
- C. Bonds issued by the firm.
- D. Common stock issued by the firm.
- E. Accounts payable of the firm.
- F. Accounts receivable of the firm.
- G. Depreciation tax shields of the firm.
- H. Deferred tax assets of the firm.
- I. Deferred tax liabilities of the firm.

*Parts A and B:* Common stock and bonds owned by the firm are assets, not liabilities or equity.

*Part C:* Bonds issued by the firm are long-term liabilities.

*Part D:* Common stock issued by the firm is equity.

*Part E:* Accounts payable of the firm are short-term liabilities.

*Part F:* Accounts receivable of the firm are assets.

*Part G:* Depreciation tax shields of the firm are not accounting entries. The tax shields mean that in the future, the interest payments are a deduction from taxable income.

*Part H:* Deferred tax assets of the firm are assets.

*Part I:* Deferred tax liabilities of the firm are liabilities, though Brealey and Myers say they are not liabilities for evaluating the weighted average cost of capital of the firm.



[Changes in debt face values and yields or other external items may affect the value of a firm and its equity.]

\*\* Exercise 16.5: Value of equity

A firm has \$25,000 of debt and \$50,000 of equity.

The risk-free rate is 5% per annum and the yield on the debt is 7% per annum.

How do each of the following affect the market value of the equity?

- A. The risk-free rate increases and the yield on the debt increases.
- B. The risk-free rate decreases and the yield on the debt decreases.
- C. The firm issues more debt and the yield on the old debt increases.
- D. The firm pays off some debt and the yield on the remaining debt decreases.
- E. Bankruptcy proceedings become more complex and the costs of bankruptcy increase.
- F. The courts revoke the limited liability of shareholders.
- G. The debt is exchanged for secured loans on the firm's assets with a 6% yield to maturity.

*Part A:* If the yield on the debt increases, the market value of the debt decreases. Most firms have more fixed liabilities (debt) than fixed assets that vary with interest rates. The value of the assets do not change materially, so the market value of the firm increases and the market value of the equity increases. (For insurers, who have investment portfolios of bonds and other fixed-income securities, this is not true. When interest rates rise, the market value of their assets declines, and their equity declines.) Brealey and Myers deal with manufacturers and other firms with much debt but little fixed-income securities as assets.

*Question:* Does the present value of the debt tax shields change?

*Answer:* If the debt is fixed and perpetual, the present value of the tax shields is  $T_c \times D$ , where  $T_c$  is the marginal tax rate and  $D$  is the market value of the debt. In other cases, the formula for the present value of the debt tax shields is more complex. In all cases, the present value does change, but the change is much less than the change in the market value of the debt itself.

*Part B:* The scenario in Part B is the reverse of the scenario in Part A.

*Part C:* Two changes may affect the market value of the equity: the greater debt and the higher yield.

Suppose the firm issues more debt but the yield on the old debt does not change. This occurs if the new debt does not materially raise the probability of bankruptcy or if the new debt is subordinate to the old debt (which is the normal relation). In perfect capital markets, with no corporate income taxes or costs of bankruptcy, the market value of the equity does not change. If the corporate tax rate is more than zero and the firm is profitable, the additional tax shields from the new debt raise the value of equity.

The new debt may raise the probability of bankruptcy and therefore raise the yield on the old debt. The market value of the old debt decreases and the market value of the equity increases, even in perfect capital markets. Brealey and Myers say that additional debt benefits stockholders and hurts existing bondholders.

*Question:* It sounds like bondholders are at the mercy of the firm and its shareholders. The firm can issue more debt and gain value at the expense of bondholders.

*Answer:* To prevent this, bond underwriters put provisions in the debt covenant preventing the firm from issuing more debt or making the additional debt subordinate to the existing debt.

*Question:* You discuss the probability of bankruptcy and then say this applies even in perfect capital markets. But perfect capital markets imply the probability of bankruptcy is zero.

*Answer:* In perfect capital markets, the costs of bankruptcy are zero. The probability of bankruptcy is almost never zero; any firm can become bankrupt unless it has no debt at all (neither short-term nor long-term).

*Part D:* The scenario in Part D is the reverse of the scenario in Part C.

*Part E:* The costs of bankruptcy are incurred by stockholders. If customers don't want to deal with a firm that might become bankrupt, the value of the firm decreases and the market value of the equity decreases. If the costs of bankruptcy increase because bankruptcy proceedings become more complex, both stockholders and bondholders lose. The bondholders lose more, since they have preference over stockholders, but everyone loses.

*Part F:* Limited liability is a benefit for stockholders. If the courts revoke this right, the market value of equity decreases.

*Part G:* In perfect capital markets, the capital structure does not affect the value of the firm, so the market value of equity does not change. Even in capital markets are not perfect and the marginal tax rate is more than zero, the market value of the debt is the same so the present value of the debt tax shields do not change, and the market value of the equity does not change.

*Question:* What happens in the corporate tax rate increases? It seems that the weighted average cost of capital decreases, implying that the value of the firm increases and the value of equity increases.

*Answer:* Suppose a firm earns \$100 million a year pre-tax with a corporate tax rate of 35%. Its after-tax earnings are \$65 million a year. If Congress changes the corporate tax rate to 50%, its after-tax earnings are \$50 million a year, so it is worth less. The effects of changing the corporate tax rate are complex. If the tax rate rises, firms must charge more for their goods and services. They must earn \$130 million pre-tax for the same \$65 million after-tax as before. Final exam problems do not ask about a change in the corporate tax rate unless they also specify the pre-tax earnings before and after the change in the tax rate.

**\*\* Exercise 16.6: Perfect capital markets**

Assume (i) capital markets are perfect; (ii) tax rates are zero on both corporate and personal income; (iii) investors and firms can borrow and lend at the risk-free interest rate.

Company ABC has 1 million shares trading at \$100 a share.

ABC has three possible income levels:

	<i>Probability</i>	<i>Income (\$000,000)</i>
<i>Poor</i>	25%	0
<i>Moderate</i>	50%	10
<i>Good</i>	25%	20

Investor XYZ owns 10,000 shares of Company ABC.

- ! Company ABC issues \$20 million of debt and pays a one-time dividend of \$20 per share.
- ! The cost of debt capital  $r_D = 5\%$  per annum. Assume this is risk-free rate.
- ! The assets of the company and the expected cash flows (except for debt payments) do not change.

- A. What is the expected return on equity before the refinancing?
- B. What is the debt-to-equity ratio after the refinancing?
- C. What is the expected return on equity after the refinancing?
- D. If investor XYZ wants no change in net income from the firm's refinancing, regardless of the firm's profits in the year, what actions should the investor take?
- E. If the firm used the bond proceeds to repurchase shares from other investors (not XYZ), what action should the investor XYZ take to keep his or her net income the same?

*Part A:* Before refinancing, the expected income is  $25\% \times \$0 + 50\% \times \$10 + 25\% \times \$20 = \$10$  million.

The expected return on equity is  $\$10 \text{ million} / (\$100 \times 1 \text{ million}) = 10\%$ .

*Part B:* After refinancing, Company ABC has \$20 million of debt and \$80 million of equity, for a  $20 / 80 = 25\%$  debt-to-equity ratio.

*Question:* Why is the equity only \$80 million instead of \$100 million?

*Answer:* The \$20 stockholder dividend reduces the value of each share from \$100 to \$80.

*Part C:* Subtract the debt payments of  $5\% \times \$20 \text{ million} = \$1 \text{ million}$  from the other income.

	<i>Probability</i>	<i>Net income (\$000,000)</i>
<i>Poor</i>	25%	-1
<i>Moderate</i>	50%	9
<i>Good</i>	25%	19

The expected income is  $25\% \times -\$1 + 50\% \times \$9 + 25\% \times \$19 = \$9$  million. Alternatively,  $\$10 \text{ million} - \$1 \text{ million} = \$9 \text{ million}$ .

The equity after refinancing is \$80 million, so the return on equity is  $9 / 80 = 11.25\%$ .

*Part D:* The investor should take the dividend he or she received of  $\$20 \times 10,000 = \$200,000$  and invest it in risk-free bonds.

*Part E:* The investor should sell 20% of his or her shares for  $\$100 \times 2,000 = \$20,000$  invest the money in risk-free bonds.

[Know the relations of the return on assets  $r_A$ , the return on equity  $r_E$ , the return on debt  $r_D$ , the beta of assets  $\beta_A$ , the beta of equity  $\beta_E$ , and the beta of debt  $\beta_D$ , in both perfect capital markets and actual capital markets.]

**\*\* Exercise 16.7: Capital structure and returns on capital**

The corporate tax rate is zero, the costs of bankruptcy are zero, and capital markets have no imperfections.

- ! The risk-free rate is 5% per annum and the market risk premium is 8% per annum.
- ! A firm with 10,000 shares of stock trading at \$100 a share has a CAPM beta of 75%.
  
- ! The firm issues \$200,000 of debt and buys back \$200,000 of stock from shareholders.
- ! The cost of debt capital  $r_D$  for \$200,000 of debt is 6% per annum.

After the refinancing (issue of debt and share repurchase):

- A. What are the share price, equity capital, and debt-to-equity ratio (D/E)?
- B. What is the return on assets  $r_A$ ?
- C. What is the return on equity  $r_E$ ?
- D. What is the beta of assets  $\beta_A$ ?
- E. What is the beta of equity  $\beta_E$ ?
- F. What is the beta of debt  $\beta_D$ ?

*Part A:* The firm repurchases shares, so the share price does not change: it stays \$100 per share. The firm buys  $\$200,000 / \$100 = 2,000$  shares, so the remaining equity is  $\$1 \text{ million} - \$200,000 = \$800,000$ . It has \$200,000 of debt and \$800,000 of equity, so the debt-to-equity ratio is 25%.

*Question:* Are these general relations that apply to all exam problems?

*Answer:* These relations apply to perfect capital markets. If the corporate tax rate is positive, the firm has tax shields from the debt, and the value of equity increases by the tax shields. In actual capital markets with a marginal tax rate more than zero, equity increases by the present value of the debt tax shields. The number of shares does not change, so the stock price increases and the debt-to-equity ratio is slightly lower.

*Part B:* Use two principles:

- ! With all equity financing, the return on assets = the return on equity =  $5\% + 75\% \times 8\% = 11\%$ . This is true whether or not capital markets are perfect.
- ! In perfect capital markets, the capital structure does not affect the return on assets.

Since debt is zero, this is also the return on assets. In perfect capital markets, refinancing doesn't change the return on assets, which stays 11%.

*Part C:* The return on debt capital is given as 6% per annum. The equation for the weighted average cost of capital (WACC) relates the return on debt capital, the return on equity capital, and the return on assets.

- ! Solve for the return on equity as  $6\% \times 20\% + Z \times 80\% = 11\% \Rightarrow$
- !  $Z = (11\% - 6\% \times 20\%) / 80\% = 12.2500\%$ .
- ! The increase in the return on equity is  $12.25\% - 11\% = 1.25\%$ .

*Take heed:* To avoid arithmetic errors, verify the result:  $20\% \times 6\% + 80\% \times 12.25\% = 11.00\%$

*Question:* What is the relation between the change in the return on equity and the debt-to-equity ratio?

*Answer:* Suppose the firm issues \$500,000 of debt and buys back \$500,000 of stock from shareholders.

- ! The new debt-to-equity ratio is 50% to 50% (1 to 1).
- ! Solve for the return on equity as  $6\% \times 50\% + Z \times 50\% = 11\% \Rightarrow$
- !  $Z = (11\% - 6\% \times 50\%) / 50\% = 16.0000\%$ .
- ! The increase in the return on equity is  $16\% - 11\% = 5\%$ .

- ! The debt-to-equity ratio in this scenario of 100% is four times as large as the debt-to-equity ratio of 25% in the previous scenario.
- ! The 5% increase in the return on equity in this scenario is four times as large as the 1.25% increase in the return on equity in the previous scenario.

Brealey and Myers state this as a general rule.

*Part D:* With all-equity financing, the return on assets = the return on equity, so the beta of assets = the beta of equity = 75% in this problem. In perfect capital markets, the capital structure does not affect the return on assets, so the beta of assets  $\beta_A$  remains 75%.

*Part E:* The beta of equity  $\beta_E$  is related to the return on equity by the CAPM equation:

$$12.25\% = 5\% + \beta_E \times 8\% \Rightarrow$$

$$\beta_E = (12.25\% - 5\%) / 8\% = 90.6250\%$$

*Take heed:* To avoid arithmetic errors, verify the result:  $5\% + 90.625\% \times 8\% = 12.25\%$

*Part F:* The beta of debt  $\beta_D$  is related to the return on equity by the CAPM equation:

$$6\% = 5\% + \beta_D \times 8\% \Rightarrow$$

$$\beta_D = (6\% - 5\%) / 8\% = 12.5000\%$$

Verify with the WACC equation:

$$20\% \times 12.5\% + 80\% \times 90.6250\% = 75.00\%$$

**\*\* Exercise 16.8: Returns and betas**

Suppose the corporate tax rate is zero and capital markets have no imperfections.

The risk-free rate is 10% and the expected market return is 18%.

A firm's debt to value ratio is 50%, its cost of debt capital  $r_D$  is 12%, and its beta of equity is 1.500.

- A. What is the firm's return on equity capital  $r_E$ ?
- B. What is the firm's return on assets  $r_A$ ?
- C. What is the firm's beta of debt  $\beta_D$ ?
- D. What is the firm's beta of assets  $\beta_A$ ?

*Part A:* The beta of equity is 1.5, so the return on equity capital from the CAPM equation is

$$10\% + 1.5 \times (18\% - 10\%) = 22.00\%.$$

*Part B:* The return on assets is  $D/V \times r_D + E/V \times r_E$ , where  $E/V = 1 - D/V$ :

$$50\% \times 12\% + 50\% \times 22\% = 17.00\%.$$

*Part C:* The beta of debt  $\beta_D$  is derived from the CAPM equation:

$$\beta_D = (r_D - r_f) / (r_m - r_f) = (12\% - 10\%) / (18\% - 10\%) = 0.250$$

*Part C:* The beta of assets  $\beta_A$  is derived from the CAPM equation:

$$\beta_D = (r_A - r_f) / (r_m - r_f) = (17\% - 10\%) / (18\% - 10\%) = 0.875$$

*Question:* How might final exam problems differ from this exercise?

*Answer:* Final exam problems might give the debt-to-equity ratio instead of the debt to value ratio. Use the relation  $E + D = A$  to convert the debt-to-equity ratio to a debt to value ratio.

Final exam problems may give the market risk premium instead of the expected market return. Use the relation risk-free rate + market risk premium = expected market return.

Final exam problems may give a non-zero corporate tax rate. Use this tax rate in the weighted average cost of capital formula.

**\*\* Exercise 16.9: Capital structure and weighted average cost of capital**

- ! With all-equity financing, a firm's opportunity cost of capital is 14%.
- ! The firm refinances to a debt-to-value ratio ( $D/V$ ) of 45%, at a cost of debt capital  $r_D$  of 9.5%.
- ! The corporate tax rate is 40%.

- A. What is the firm's cost of equity capital  $r_E$  after the refinancing?
- B. What is the firm's weighted average cost of capital (WACC) after the refinancing?

*Part A:* If the corporate tax rate were zero, the weighted average cost of capital would be 14%, the same as with all-equity financing.

After refinancing, the debt-to-value ratio ( $D/V$ ) = 45% and the equity-to-value ratio ( $E/V$ ) =  $1 - 45\% = 55\%$ .

Solve for the return on equity capital as

$$45\% \times 9.5\% + 55\% \times Z = 14\% \Rightarrow$$
$$Z = (14\% - 45\% \times 9.5\%) / 55\% = 17.68\%$$

*Part B:* The weighted average cost of capital after refinancing is

$$45\% \times 9.5\% \times (1 - 40\%) + 55\% \times Z = 12.29\%$$



**\*\* Exercise 16.10: Capital structure and weighted average cost of capital**

- ! With all-equity financing, a firm's opportunity cost of capital is 12%.
- ! The firm refinances to a debt-to-value ratio (D/V) of 40%, at a cost of debt capital  $r_D$  of 6%.
- ! The corporate tax rate is 41.5%.

- A. What is the firm's cost of equity capital  $r_E$  after the refinancing?
- B. What is the firm's weighted average cost of capital (WACC) after the refinancing?

*Part A:* If the corporate tax rate were zero, the weighted average cost of capital would be 12%, the same as with all-equity financing.

After refinancing, the debt-to-value ratio (D/V) = 40% and the equity-to-value ratio (E/V) =  $1 - 40\% = 60\%$ .

Solve for the return on equity capital as

$$40\% \times 6\% + 60\% \times Z = 12\% \Rightarrow$$
$$Z = (12\% - 40\% \times 6\%) / 60\% = 16.00\%$$

*Part B:* The weighted average cost of capital after refinancing is

$$40\% \times 6\% \times (1 - 41.5\%) + 60\% \times 16\% = 11.00\%$$