

## Corporate Finance, Module 17 (Depreciation Problems)

### Practice Problems on Depreciation

(The attached PDF file has better formatting.)

{Note: The following two part problem shows the effect of depreciation schedules on the net present value of a project. You need not know the rules of different depreciation schedules for the final exam; the exam specifies the depreciation schedule to use. Given the depreciation schedule and the cash flows, you should derive the net present value.}

#### Exercise 17.1: Part 1 – Depreciation Schedules

An all equity financed project requires an initial investment of \$16.5 million and generates *pre-tax* net cash flows of \$3.5 million each year for ten years.

The initial investment at time 0 is \$1.5 million for expenses and \$15 million for equipment. The expenses are written off immediately for tax purposes. The equipment is capitalized and depreciated evenly (straight-line) over ten years and has no salvage value.

Shareholders require a 12% after-tax return, and the corporate tax rate is 35%.

- A. What is the pre-tax cash outflow at time 0? (Expenses plus the cost of machinery.)
- B. What is the after-tax cash outflow at time 0? The expenses are written off when they are incurred, so they provide a tax refund; they are not amortized over the life of the project.
- C. What is the depreciation each year? (\$15 million equipment over 10 years.)
- D. What is the taxable income each year? (Cash of \$3.5 million *minus* the depreciation.)
- E. What is the tax liability each year? (Taxable income times the tax rate.)
- F. What is the after-tax income each year? (Taxable income minus the tax liability.)
- G. What is the after-tax cash flow each year? (After-tax income plus depreciation.)
- H. What is the present value of the annual after-tax cash flow, using the 12% after-tax return.)
- I. What is the net present value of the project? (The present value of the after-tax cash inflows minus the after-tax cash outflow in year 0.)

#### Solution 17.1:

*Part A:* The pre-tax cash outflow at time 0 is \$15 million for the machinery and \$1.5 million for expenses, or \$16.5 million.

*Part B:* The equipment is depreciated over 10 years, so it provides no tax relief in year 0. The \$1.5 million of expenses cause taxable income for year 0 of  $-\$1.5$  million, so the tax liability is a refund of  $\$1.5 \text{ million} \times 35\% = \$0.525$  million. The after-tax cash outflow is  $\$16.5 \text{ million} - \$0.525 \text{ million} = \$15.975$  million.

*Part C:* The depreciation each year is  $\$15 \text{ million} / 10 \text{ years} = \$1.5$  million a year.

*Part D:* The taxable income each year is the pre-tax cash flow of \$3.5 million minus the depreciation of \$1.5 million each year, or \$2 million.

*Part E:* The tax liability is the taxable income times the tax rate of 35%, or  $\$2 \text{ million} \times 35\% = \$700,000$ .

*Part F:* The after-tax income is pre-tax income minus the tax liability, or  $\$2 \text{ million} - \$700,000 = \$1.3$  million.

*Part G:* The after-tax cash inflow is the after-tax income of \$1.3 million plus the depreciation of \$1.5 million = \$2.8 million.

*Illustration:* Suppose the net cash flow is \$2.8 million and depreciation is \$1.5 million. GAAP income is \$2.8 million – \$1.5 million = \$1.3 million. We add back depreciation to get the net cash flow.

*Part H:* The present values of each year's after-tax cash flow are shown in the table below.

*Part I:* The present value of the after-tax net cash flows is  $\$2.80 \text{ million} \times (1-v^{10})/i = \$15.82 \text{ million}$ . The after-tax initial investment is  $\$15 \text{ million} + \$1.5 \text{ million} \times (1 - 35\%) = \$15.98 \text{ million}$ . The net present value of the project is  $\$15.821 \text{ million} - \$15.98 \text{ million} = -\$0.16 \text{ million}$ . The project has a negative after-tax net present value and is not worthwhile.

Year	Percentage	Depreciation	Taxable Income	After-Tax Income	After-Tax Cash Flow	Present Value
1	10.00%	1.500	2.000	1.300	2.800	2.500
2	10.00%	1.500	2.000	1.300	2.800	2.232
3	10.00%	1.500	2.000	1.300	2.800	1.993
4	10.00%	1.500	2.000	1.300	2.800	1.779
5	10.00%	1.500	2.000	1.300	2.800	1.589
6	10.00%	1.500	2.000	1.300	2.800	1.419
7	10.00%	1.500	2.000	1.300	2.800	1.267
8	10.00%	1.500	2.000	1.300	2.800	1.131
9	10.00%	1.500	2.000	1.300	2.800	1.010
10	10.00%	1.500	2.000	1.300	2.800	0.902
Tot	100.00%	15.000	20.000	13.000	28.000	15.821

### Exercise 17.2: Part 2 – Depreciation Schedules (continued)

An all equity financed project requires an initial investment of \$16.5 million and generates *pre-tax* net cash flows of \$3.5 million each year for ten years.

The initial investment at time 0 is \$1.5 million for expenses and \$15 million for equipment. The expenses are written off immediately for tax purposes. The equipment is capitalized at purchase, and the firm uses *sum of the year digits depreciation* over ten years, after which it has no salvage value.

Shareholders require a 12% after-tax return, and the corporate tax rate is 35%.

- A. What is the pre-tax cash outflow at time 0? (Expenses plus the cost of machinery.)
- B. What is the after-tax cash outflow at time 0? The expenses are written off when they are incurred, so they provide a tax refund; they are not amortized over the life of the project.
- C. What is the depreciation each year? (\$15 million equipment over 10 years.)
- D. What is the taxable income each year? (Cash of \$3.5 million *minus* the depreciation.)
- E. What is the tax liability each year? (Taxable income times the tax rate.)
- F. What is the after-tax income each year? (Taxable income minus the tax liability.)
- G. What is the after-tax cash flow each year? (After-tax income plus depreciation.)
- H. What is the present value of the annual after-tax cash flow, using the 12% after-tax return.)
- I. What is the net present value of the project? (The present value of the after-tax cash inflows minus the after-tax cash outflow in year 0.)

*Question:* What is sum of the years depreciation?

*Answer:* In this exercise, the equipment has a ten year life. The sum of the digits  $\{1 + 2 + \dots + 9 + 10\} = 55$ . The depreciation is  $10 / 55 = 18.18\%$  the first year,  $9 / 55 = 16.36\%$  the second year, ..., and  $1 / 55 = 1.82\%$  the tenth year.

Solution 17.2:

*Part A:* The pre-tax cash outflow at time 0 is \$15 million for the machinery and \$1.5 million for expenses, or \$16.5 million.

*Part B:* The equipment is depreciated over 10 years, so it provides no tax relief in year 0. The \$1.5 million of expenses cause taxable income for year 0 of  $-\$1.5$  million, so the tax liability is a refund of  $\$1.5 \text{ million} \times 35\% = \$0.525$  million. The after-tax cash outflow is  $\$16.5 \text{ million} - \$0.525 \text{ million} = \$15.975$  million.

*Part C:* The depreciation each year is shown in the table below, using the sum of the years method. The total depreciation is still \$15 million, but the depreciation is now *accelerated*: more depreciation is written off in the early years and less in the later years. The depreciation is \$2.727 million the first year and \$0.273 million the tenth year.

*Part D:* The taxable income each year is the pre-tax cash flow of \$3.5 million minus the depreciation. The taxable income ranges from \$0.773 million the first year to \$2.337 million the tenth year.

*Part E:* The tax liability is the taxable income times the tax rate of 35%.

*Part F:* The after-tax income is pre-tax income minus the tax liability.

*Part G:* The after-tax cash inflow is the after-tax income plus the depreciation, ranging from \$3.23 million the first year to \$2.37 million the tenth year.

*Part H:* The present values of each year's after-tax cash flow are shown in the table below.

<i>Year</i>	<i>Digits</i>	<i>Percentage</i>	<i>Depreciation</i>	<i>Taxable Income</i>	<i>After-Tax Cash Flow</i>	<i>Present Value</i>
1	10	18.18%	2.727	0.773	3.230	2.884
2	9	16.36%	2.455	1.045	3.134	2.498
3	8	14.55%	2.182	1.318	3.039	2.163
4	7	12.73%	1.909	1.591	2.943	1.870
5	6	10.91%	1.636	1.864	2.848	1.616
6	5	9.09%	1.364	2.136	2.752	1.394
7	4	7.27%	1.091	2.409	2.657	1.202
8	3	5.45%	0.818	2.682	2.561	1.034
9	2	3.64%	0.545	2.955	2.466	0.889
10	1	1.82%	0.273	3.227	2.370	0.763
	55	100.00%	15.000	20.000	28.000	16.314

*Part I:* The present value of the after-tax net cash flow is \$16.314 million, and the net present value of the project is \$16.314 million – \$15.98 million = \$0.33 million. The project has a positive after-tax net present value and it is worthwhile.

Question 17.3: Tax Depreciation

A firm acquires a subsidiary, creating goodwill of \$100 million, which is amortized (pro-rata) over Z years, ranging from 0 years (immediate write-off) to 20 years. The depreciation (amortization) period differs for tax purposes vs GAAP accounting statements.

Which of the following values for Z is most advantageous to the firm?

- |    | <u>Tax</u> | <u>GAAP</u> |
|----|------------|-------------|
| A. | 0 yrs      | 20 yrs      |
| B. | 10 yrs     | 10 yrs      |
| C. | 10 yrs     | 20 yrs      |
| D. | 20 yrs     | 10 yrs      |
| E. | 20 yrs     | 20 yrs      |

Why do firms desire short depreciation (amortization) periods for tax purposes and long period for GAAP (book accounting) statements?

Answer 17.3: A

Immediate tax depreciation gives the largest present value tax benefit and the highest net present value for the firm (and its projects). Firms generally desire long GAAP depreciation (amortization) periods, so that their balance sheets show high values for the firm.

#### Question 17.4: Depreciation

Suppose the corporate tax rate is 35%. Which of the following statements are true regarding tax depreciation?

- A. The more costs that can be depreciated (vs being written off when they are incurred), the greater the NPV of a project.
- B. The longer the depreciation schedule, the greater the NPV of a project.
- C. If the government increases the corporate tax rate, many costs are depreciated, and the earnings before interest and taxes remain the same, the project NPV increases.
- D. Accelerating the depreciation schedule increases the NPV of a project.
- E. None of A, B, C, or D is true.

Answer 17.4: D

*Statement A:* If an expense is written off when it is incurred instead of being depreciated, the firm receives the tax benefit *up front*. This tax benefit has a greater present value than the tax benefits from depreciation. Depreciating a cost reduces the NPV.

*Statement B:* Lengthening the depreciation schedule decelerates the tax benefits; with a longer depreciation schedule, the tax benefits have the same nominal value but a lower present value, so the present value of the project decreases.

*Statement C:* If the earnings before interest and taxes remain the same and the corporate tax rate increases, the earnings after-tax earnings decrease. Although the tax shield slightly increases, its effect is overwhelmed by the decrease in after-tax earnings.

*Statement D:* Accelerating (shortening) the depreciation schedule speeds up the receipt of the tax benefits, and the present value of the project increases.

Exercise 17.5: Depreciation Schedule

(Adapted from question 24 of the Spring 2001 Course 2 examination)

An asset with a depreciation basis of \$100,000 and no salvage value is depreciated by the following schedule:

<u>Year</u>	<u>Percent</u>
1	33.33%
2	44.45%
3	14.81%
4	7.41%

The marginal tax rate is 35% and the pretax borrowing rate is 12%.

- What is the depreciation in each year?
- What is the depreciation tax shield in each year?
- What discount rate should be used to compute the present value of the tax shields?
- What is the present value of the tax shields in each year?
- What is the total present value of the tax shields?

Solution 17.5:

*Part A:* The depreciation each year is the depreciation percentage times the \$100,000 cost of the asset. For year 1, the depreciation is  $33.33\% \times \$100,000 = \$33,330$ .

*Part B:* The tax shield is the depreciation times the corporate tax rate of 35%. For year 1, the tax shield is  $\$33,330 \times 35\% = \$11,666$ .

*Part C:* The discount rate is the after-tax interest rate (after-tax yield). In the exercise, the after-tax interest rate is  $12\% \times (1 - 35\%) = 7.80\%$ .

*Part D:* For year 1, the present value of the tax shield is  $\$11,666 / 1.0780 = \$10,822$ .

Year	Depreciation	Tax Shield	Present Value
1	\$33,330	\$11,666	\$10,821
2	\$44,450	\$15,557	\$13,388
3	\$14,810	\$5,184	\$4,138
4	\$7,410	\$2,594	\$1,920
Total	\$100,000	\$35,000	\$30,267

*Part E:* The total present value of the four years' depreciation is \$30,267.

*Question:* Why do we use the *after-tax* capitalization rate?

*Answer:* Suppose we invest \$30,267 at 12% per annum pre-tax at time 0, which is a 7.80% after-tax investment yield. We compare the accumulated investment with the tax shields.

Year	Funds	Accumulated	Depreciated	Tax	Funds – Tax
1	\$30,267	\$32,628	\$33,330	\$11,666	\$20,962

2	\$20,962	\$22,597	\$44,450	\$15,557	\$7,040
3	\$7,040	\$7,589	\$14,810	\$5,184	\$2,405
4	\$2,405	\$2,593	\$7,410	\$2,594	\$0
Total			\$100,000	\$35,000	

With a 12% pre-tax investment yield, or a 7.80% after-tax investment yield, \$30,267 at time 0 just equals the present value of the tax shields.

*Question:* When we discount insurance losses or insurance benefits, we use *pre-tax* discount rates, not after-tax discount rates.

*Answer:* We discount pre-tax cash flows at pre-tax discount rates and after-tax cash flows at after-tax discount rates. Insurance losses are a pre-tax expense, so we use pre-tax discount rates. Depreciation tax shields are an after-tax cash flow, so we use after-tax discount rates.