Corporate Finance, Module 19: Adjusted Present Value

Subsidized Loans Practice Problems

(The attached PDF file has better formatting.)

Exercise 19.1: Subsidized Loan

A firm purchases a $50,000 truck. The seller lends the firm $50,000 for five years, with interest payments of 5% at the end of each year, and the $50,000 principal repaid at the end of year 5.

The firm would pay 15% to borrow from a bank. The marginal corporate tax rate is 35%.

A. What are the pre-tax coupon payments on a 15% loan?
B. What are the pre-tax coupon payments on the 5% loan?
C. What are the after-tax coupon payments on a 15% loan?
D. What are the after-tax coupon payments on the 5% loan?
E. What is the difference in each year's coupon payments?
F. What is the after-tax borrowing rate?
G. What is the present value of this difference in coupon payments?

Solution 19.1:

Part A: The annual interest payments on a 15% bank loan would be 15% × $50,000 = $7,500.

Part B: The annual interest payments on this loan are 5% × $50,000 = $2,500.

Part C: The after-tax payments are $7,500 × (1 – 35%) = $4,875 on the bank loan.

Part D: The after-tax payments are $2,500 × (1 – 35%) = $1,625 on the subsidized loan.

Question: The interest payments are tax deductible. If income is taxed, then pre-tax income differs from after-tax income; $1,000 of pre-tax income is $650 of after-tax income. If a cash flow is not taxed, the pre-tax amount should be the same as the after-tax amount.

Answer: The terms are confusing. The after-tax payments mean: "How much more after-tax income would the firm need to make the interest payments in all sources of capital, including debt, were fully taxable?" If the firm has $4,875 more of after-tax income, its pre-tax income must be $4,875 / (1 – 35%) = $7,500 higher, which is sufficient to pay the debt coupon.

Part E: The difference in after-tax payments is $4,875 – $1,625 = $3,250 per annum.

Part F: The firm's after-tax borrowing rate is 15% × (1 – 35%) = 9.75%. We use the 15% bank borrowing rate, not the 5% subsidized rate.

Part G: The value of the loan is $3,250 for five years capitalized at the after-tax yield of 9.75% = $12,399.15 ≈ $12,400.

ALTERNATIVE CALCULATION: The net present value of the loan is the present value of the cash inflows minus the present value of the cash outflows.

A loan at Z% per annum has a value of zero if interest rates do not change. Algebraically,

$$1 - \sum_{n=1}^{n=N} z/(1+z)^n - 1/(1+z)^N = 0$$

Illustration: Suppose the principal is \$50,000, the term is five years, and the interest rate is 15% per annum with annual compounding. The interest each year is \$7,500, and the present value of the loan is

$$\$50{,}000 - \sum_{n=1}^{n=5} \$7{,}500/1.15^n - \$50{,}000/1.15^5 = 0$$

The tax shields do not affect the present value of the loan. The before tax value of the loan is zero and the after-tax value of the loan is zero. But if we consider the after-tax payment on the loan in terms of after-tax dollars, the after-tax payments are \$7,500 × (1 – tax rate) = \$4,875 each year. To get a zero net present value for the loan, we use 15% × (1 – tax rate) = 9.75% as the capitalization rate.

For the present value of the subsidized loan, we examine the net present value at the after-tax payments at a 9.75% capitalization rate. The payments are \$50,000 × 5% × (1 – 35%) = \$1,625 each year. The value of the subsidized loan is

$$\$50{,}000 - \sum_{n=1 \text{ to } 5} \$1{,}625/1.0975^n - \$50{,}000/1.0975^5 = \$12{,}399.15.$$

A CHECK

We examine whether \$12,399.15 and the unsubsidized loan is worth the subsidized loan. Suppose Firm A has \$12,399.15 invested at 15% pre-tax and the 15% bank loan; Firm B has the 5% loan.

			Firm A		*Firm B*		
Cash	*Interest Income*	*After-tax Interest Income*	*Debt Coupon*	*After-tax Debt Coupon*	*Debt Coupon*	*After-tax Debt Coupon*	*Debt Coupon Difference*
12,399.15	1,859.87	1,208.92	7,500.00	4,875.00	2,500.00	1,625.00	3,250.00
10,358.07	1,553.71	1,009.91	7,500.00	4,875.00	2,500.00	1,625.00	3,250.00
8,117.98	1,217.70	791.50	7,500.00	4,875.00	2,500.00	1,625.00	3,250.00
5,659.48	848.92	551.80	7,500.00	4,875.00	2,500.00	1,625.00	3,250.00
2,961.28	444.19	288.72	7,500.00	4,875.00	2,500.00	1,625.00	3,250.00
0.01							

After five years, the difference between the two firms is 1¢ (a rounding error).

Question: Should we know these different methods?

Answer: The first method is needed for this course. Many candidates don't grasp the intuition at first, and have trouble explaining why the solution is correct. The alternative method and the check help you understand the intuition.

Exercise 19.2: Subsidized Loan

(Adapted from question 4 of the Spring 1999 Course 2 examination)

A firm finances the purchase of a $100,000 machine with a three year loan from the manufacturer at annual coupon payments of 6%. The $100,000 principle is repaid at the end of year 3 with the last interest payment. The firm would pay 12% to borrow from a bank. The marginal tax rate is 35%.

A. What are the pre-tax coupon payments on a 12% loan?
B. What are the pre-tax coupon payments on the 6% loan?
C. What are the after-tax coupon payments on a 12% loan?
D. What are the after-tax coupon payments on the 6% loan?
E. What is the difference in each year's coupon payments?
F. What is the after-tax borrowing rate?
G. What is the present value of this difference in coupon payments?

Solution 19.2:

Part A: The annual interest payments on a 12% bank loan would be 12% × $100,000 = $12,000.

Part B: The annual interest payments on this loan are 6% × $100,000 = $6,000.

Part C: The after-tax payments are $12,000 × (1 – 35%) = $7,800 on the bank loan.

Part D: The after-tax payments are $6,000 × (1 – 35%) = $3,900 on the subsidized loan.

Part E: The difference in after-tax payments is $7,800 – $3,900 = $3,900 per annum.

Part F: The firm's after-tax borrowing rate is 12% × (1 – 35%) = 7.80%. We use the 12% bank borrowing rate, not the 6% subsidized rate.

Part G: The value of the loan is $3,900 for three years capitalized at the after-tax yield of 7.80% = $10,087.06.

Question: What should we focus on in these problems?

Answer: We take the present value of the after-tax difference in the cash flows at the after-tax capitalization rate. The pre-tax capitalization rate is the yield on the unsubsidized loan.

Question: If the value of the loan to the purchaser is $10,000, is the cost of the loan to the supplier also $10,000? How does the tax shield affect the supplier's cost? The gain to the purchaser is reduced by the smaller present value of the tax shields. Since the tax shields to the buyer are smaller, the IRS gains. Can we infer that the cost to the supplier is greater than the savings to the buyer?

Answer: Suppose a buyer pays $100,000 for new equipment which is written off immediately as an expense for tax purposes. For simplicity, assume the seller has no cost of producing the equipment. The seller's taxable income rises by $100,000, and the buyer's taxable income declines by $100,000; the IRS gets no net revenue.

Suppose the buyer takes a loan from a bank at 15% interest. The interest payment raises the bank's taxable income and it reduces the buyer's taxable income. The IRS has no net revenue.

Some people think of the tax deductibility of interest payments as a subsidy by the IRS. This is not true; it simply prevent double taxation of the same income. We are so used to the IRS taxing us two or three times on the same income that if we are taxed only once, we presume that the IRS is subsidizing the transaction.