Corporate Finance Mod 21: Option deltas, practice problems

** Exercise 21.1: Call and Put Deltas

One year put and call options are trading on a stock with a $75 exercise price.

! At 10:00 am, the stock trades at $70, the **call** option is worth $12.80, and the **put** option is worth $10.30.
! At 1:00 pm, the stock trades at $71, the **call** option is worth $13.20, and the **put** option is worth $9.70.

A. What is the value of the call option delta?
B. What is the value of the put option delta?

Part A: The option delta is the partial derivative of the option price with respect to the stock price. For a $1 increase in the stock price from 10:00 am to 1:00 pm, the call option increases by 40¢ and the put option decreases by 60¢. The call delta is ($13.20 – $12.80) / ($71.00 – $70.00) = $0.40 / $1.00 = 40%.

Part B: The put delta is ($9.70 – $10.30) / ($71.00 – $70.00) = –$0.60 / $1.00 = –60%.

** Exercise 21.2: Option Deltas

A stock trades now at $80, and it may move up by 25% or down by 20% over the next year. One year European put and call options are trading with a strike price of $90.

A. What is the delta of the *call* option?
B. What is the delta of the *put* option?

Part A: The delta of an option is the difference in the value of the option divided by the difference in the value of the stock.

We determine the price of the stock and of the call option in one year in the up and down scenarios, and take the change in the value of the call option divided by the change in the value of the stock.

The stock price either moves up by 25% to $100 or down by 20% to $64.

- If the stock price moves up to $100, the call option is worth $100 – $90 = $10.
- If the stock price moves down to $64, the call option is worth nothing, since the stock price is less than the exercise price.

The change in the stock price from up to down movements is $100 – $64 = $36. The change in the value of the call option is $10 – $0 = $10. The call option delta is

$$(\$10 - \$0) / (\$100 - \$64) = 0.278$$

A call option becomes more valuable when the price of the stock increases, so the delta is positive.

Part B: A put option becomes less valuable when the price of the stock increases, so the delta is positive.

- If the stock price moves up to $100, the put option is worth zero, since the stock price exceeds the exercise price.
- If the stock price moves down to $64, the put option is worth $90 – $64 = $26.

The change in the stock price from up to down movements is $100 – $64 = $36. The change in the value of the put option is $0 – $26 = –$26. The put option delta is

$$(\$0 - \$26) / (\$100 - \$64) = -0.722$$

The delta of the put option = 1 – the delta of the call option.

** Exercise 21.3: Option Deltas

Three month put and call options are trading with an exercise price of $80.

A. If the stock price increases without bound, what happens to the delta of the call option?
B. If the stock price decreases to zero, what happens to the delta of the call option?
C. If the stock price increases without bound, what happens to the delta of the put option?
D. If the stock price decreases to zero, what happens to the delta of the put option?

Part A: As the stock price increases without bound, the probability of the price declining below the exercise price before the expiration date tends to zero. The call option will almost certainly be exercised, so the value of the call option approaches the stock price minus the present value of the exercise price. If the stock price rises a dollar, the value of the call option rises a dollar. The limit of the delta of the call option as the stock price goes to infinity is one.

Part B: If the stock price decreases to zero, investors believe the stock price will not rise above the exercise price. If investors thought the stock price might rise next week, they would not bid the price down to zero now.

Suppose the price falls to 1¢, and investors think the probability of the price rising above $80 is 1 in ten million. The call option is worth money only if the stock price rises above $80, so its value might be 0.00001¢. If the stock price rises to 2¢, the value of the call option might rise to 0.00002¢. The delta of the call option is

$$(0.00002¢ – 0.00001¢) / (2¢ – 1¢) = 0.00001 \approx 0$$

Question: The percentage change in the value of the call option is large. Do we consider this?

Answer: The delta of the option is the dollar change, not the percentage change.

Part C: As the stock price increases without bound, the probability of the price declining below the exercise price before the expiration date tends to zero. The put option will almost certainly not be exercised, so the value of the put option approaches zero. If the stock price rises another dollar, the value of the put option falls a tiny bit more. The limit of the delta of the put option as the stock price goes to infinity is zero.

Part D: If the stock price decreases to zero, investors believe the stock price will not rise above the exercise price. If investors thought the stock price might rise next week, they would not bid the price down to zero now.

Suppose the stock price is 1¢, and investors think it won't change much before the option expires. The put option is worth about $80 – 1¢ = $99.99. If the stock price rises to 2¢, the value of the put option might decline to $99.98. The delta of the put option is

$$($99.98 – $99.99) / (2¢ – 1¢) \approx –1.$$

** Exercise 21.4: Option Deltas

A stock price is now $100, and it may move up by 20% or down by 20% each **three** month period. European put and call options trade on this stock with a strike price of $112 and a term to maturity of **six** months.

A. If the stock moves up to $120 in the first three months, what is the *call* option delta at that point?
B. If the stock moves down to $80 in the first three months, what is the *call* option delta at that point?
C. If the stock moves up to $120 in the first three months, what is the *put* option delta at that point?
D. If the stock moves down to $80 in the first three months, what is the *put* option delta at that point?

Part A: If the stock trades at $120 after three months, it will either move up to $144 or down to $96 in the next three months.

- If the stock price is $144, the call option is worth $144 – $112 = $32.
- If the stock price moves down to $96, the call option is worth zero, since the stock price is less than the exercise price.

The change in the stock price from up to down movements is $144 – $96 = $48. The change in the value of the call option is $32 – $0 = $32. The call option delta is

$$(32 – 0) / (144 – 96) = 0.66667$$

Part B: If the stock trades at $80 after three months, it will either move up to $96 or down to $64 in the next three months. In both cases, the call option is worth nothing, so the delta of the call option is zero.

Part C: If the stock price is $120 at 3 months, it moves up 20% to $144 or down 20% to $96 in the next three months.

- If the stock price is $144, the put option is worth zero, since the stock price exceeds the exercise price.
- If the stock price moves down to $96, the put option is worth $112 – $96 = $16.

The change in the stock price from up to down movements is $144 – $96 = $48. The change in the value of the put option is $0 – $16 = –$16. The put option delta is

$$(0 – 16) / (144 – 96) = –0.33333$$

Part D: If the stock trades at $80 after three months, it will either move up to $96 or down to $64 in the next three months.

- If the stock price up up to $96, the put option is worth $112 – $96 = $16.
- If the stock price moves down to $64, the put option is worth $112 – $64 = $48.

The change in the stock price from up to down movements is $96 – $64 = $32. The change in the value of the put option is $16 – $48 = –$32. The put option delta is

$$(16 – 48) / (96 – 64) = –1.00000$$

** Exercise 21.5: Stock Price and Deltas

Three month European call and put options on ABC Corp. sell at an exercise price (strike price) of $65. On January 1, at 10:00 am, the stock price is $68.

- At 10:00 am, the call option is worth $10 and the put option is worth $2.
- At 11:00 am, the call option is worth $8 and the put option is worth $4.

If the risk-free interest rate has not changed between 10:00 am and 11:00 am.

A. What is the stock price at 11:00 am?
B. What is the delta of the call option?
C. What is the delta of the put option?

Part A: Let the stock price be S. By the put call parity relation: *call + present value of exercise price = put + stock price*, so a portfolio of 1 call minus 1 put = stock price – the present value of the exercise price.

- At 10:00 am, 1 call – 1 put = $10 – $2 = $8
- At 11:00 am, 1 call – 1 put = $8 – $4 = $4

Between 10:00 am and 11:00 am, the value of 1 call – 1 put declines by $4, so the stock price must have declined by $4. The new stock price is $68 – $4 = $64.

Part B: The delta of the call option is ($8 – $10) / ($64 – $68) = +0.500.

Part C: The delta of the put option is ($4 – $2) / ($64 – $68) = –0.500.

Call option deltas range from 0 to 1. Put option deltas range from –1 to 0.