Corporate Finance, Module 21: “Option Valuation”

Homework Assignment

(The attached PDF file has better formatting.)

Binomial Tree Pricing Method

A firm’s current share price is $80; one year from now, its share price will either fall to $76 or rise to $92. The risk-free rate is 5%, and one-year European call and put options on the stock have an exercise price of $85.

A. What is the upward movement if the stock price rises to $92? (Express this as a factor of $92 / $80.) Call this upward movement by the symbol *U*.
B. What is the value of the *call* option at its maturity if the stock price rises to $92? (The call option gives the investor the right to *buy* the stock for the strike price of $85. How much would the investor pay to buy a stock for $85 if its price is $92?) Call this price Call^+, or the value of the call option at maturity if the stock price rises.
C. What is the value of the *put* option if the stock price rises to $92? (The put option gives the investor the right to *sell* the stock for the strike price of $85. How much would the investor pay to sell a stock for $85 if its price is $92?) Call this price Put^+, or the value of the put option at maturity if the stock price rises.
D. What is the downward movement if the stock price falls to $76? Call this downward movement by the symbol *D*.
E. What is the value of the call option if the stock price falls to $76? ($\text{Call}^-$)
F. What is the value of the put option if the stock price falls to $76? ($\text{Put}^-$)
G. The *actual probability* of a rise in the stock price is not relevant for options pricing. We discuss it here to differentiate it from the risk-neutral probability. Suppose the expected return on the stock is 12% per annum. The stock has only two possible values at the end of the year, $92 or $76. If the probability of rising to $92 is P, it must be that $P \times U + (1 - P) \times D = 1.120$. We solved for U and D earlier; now solve for P.
H. What is the *risk-neutral probability* of a rise in the stock price? If all investors are risk-neutral, the expected return from the stock is 5% per annum, not 12% per annum. To determine the risk-neutral probability, solve $P \times U + (1 - P) \times D = 1.050$. We use this value of P in the remaining parts of this homework assignment.
I. We used U and D as factors; we can also express them as percentage returns. If U = $92 / $80 – 1 and D = $76 / $80 – 1, then $P \times U + (1 - P) \times D = 5\%$. (There is nothing to write for this part; it is informative.)
J. What is the expected value of the call option at its maturity in a risk-neutral world? (We solved for the values of the call option at its maturity if the stock price moves up or down. Using the value of P, solve for the expected value of the call option at its maturity in the risk-neutral world: $P \times \text{Call}^+ + (1 - P) \times \text{Call}^-$)
K. What is the present value of the call option? (In a risk-neutral world, all discounting is done at the risk-free interest rate. Discount the value obtained above at a 5% rate.)
L. What is the expected value of the put option at expiration in a risk-neutral world?
M. What is the present value of the put option?
N. Verify that the put call parity relation holds. Using the present values of the call option and put option, show that call + PV(exercise price) = put + stock price.