Corporate Finance, Module 22: “Real Options”

Practice Problems

(The attached PDF file has better formatting.)

These practice problems shows the variety of applications for real options. The last problem is a realistic example that occurs continually in American business; *this example is necessary to answer the homework assignment for this module*. The questions on the final exam are not as complex as the problems here.

Exercise 22.1: Baseball Bonus

An athlete receives $100,000 for every home run he hits over 20, subject to a maximum of $3 million. Construct a set of options that model the bonus.

Solution 22.1: Consider a stock whose value is the number of home runs. The bonus contract is a bull spread on the stock. The long call option has a strike price of 20 and the short call option has a strike price of 50. The bonus is $100,000 × ($call_{20}$ – $call_{50}$).

Exercise 22.2: Reinsurance as Options

An insurer has purchased an excess-of-loss reinsurance contract from a reinsurer providing coverage for $10 million excess of $20 million: the reinsurer pays up to, but no more than, $10 million beyond the initial $20 million retained by the ceding company. Using the ground-up loss as the underlying asset, model this reinsurance contract as a combination of options.

Solution 22.2:

The ceding company has bought a call option with a strike price of $20 million and has sold a call option with a strike price of $30 million.

- If the ground-up loss is less than $20 million, neither option is exercised.
- If the ground-up loss is more than $20 million and less than $30 million, the first call option is exercised: the ceding company receives the loss minus $20 million.
- If the ground-up loss is more than $30 million, both options are exercised: the ceding company receives the loss minus $20 million and it pays the loss minus $30 million, leaving it a $10 million gain.

Exercise 22.3: Options as a Loan

A firm owns a vacant lot with a book value of $50,000. The firm finds a buyer willing to pay $200,000 for the lot, but it must give the buyer a put option to sell the lot back for $200,000 at the end of two years. Moreover, it agrees to pay the buyer $40,000 for a call option to repurchase the lot for $200,000 at the end of two years.

A. What happens if the lot is worth more than $200,000 at the end of two years? What if it is worth less than $200,000?
B. In effect, the firm has borrowed money from the buyer. What is the (annually compounded) interest rate per year on the loan?

Solution 22.3:

Part A: If the lot is worth more than $200,000 at the of two years, the firm that sells the lot exercises its call option, pays the buyer $200,000, and takes back the lot. If the lot is worth less than $200,000, the buyer exercises its put option, gives the lot back to the original owner, and receives $200,000.

Part B: The lot is in possession of the buyer for two years, at which time it reverts back to the original owner, since either the call option or the put option will be exercised. The seller now receives $200,000 – $40,000 from the buyer, and he pays $200,000 in two years. The implicit interest on the loan of $160,000 is $40,000. This is a two year loan, so the implicit interest rate r satisfies $160,000 × $(1+r)^2$ = $200,000 ⇛ $r = (1.25)^{½} – 1$ = 11.80%.

This exercise assumes that ownership of the lot for the two years has no benefits or costs. One can’t use the vacant lot for parking, storage, or other activities, and there are no taxes imposed on ownership of the lot.

Exercise 22.4: Convention Center

You are considering purchasing a convention center. You have the option to buy now or to delay your decision for one year. The purchase price will be $500 million, regardless of when it is bought. You determine the current value of the project to be $600 million. If the demand for the center is high in the first year, the cash flow will be $80 million and the value rises to $800 million. If the demand is low, the cash flow will be $40 million and the value falls to $450 million. Assume that investors are indifferent to risk and that the annual risk-free rate is 5%.

A. Calculate the value of the call option to wait one year before purchasing the convention center.
B. Should you proceed with the purchase now or delay your decision for one year?

Solution 22.4:

We determine the risk-neutral probability that the demand for the center is high the first year. Investors are indifferent to risk and the risk-free rate is 5%. The value of the center next year is $800 + $80 = $880 million if demand is high and $450 + $40 = $490 million if demand is low. The upward movement is $880 / $600 = 146.67% and the downward movement is $490 / $600 = 81.67%. The probability that demand will be high is

$$(1.05 - 0.8167) / (1.4667 - 0.8167) = 35.90\%.$$

The strike price of this call option (to wait one year before investing) is $500 million. We purchase the convention center next year if demand is high, so the value of the option is

$$35.90\% \times (\$800 - \$500) / 1.05 = \$102.57 \text{ million.}$$

The net present value of buying the convention center now is $600 – $500 = $100 million. It is worth waiting and buying the center next year only if demand is high.

Exercise 22.5: Project Pilot

A manufacturing company is looking at a potential new product. The cost of the one-year pilot portion of the project is $100,000. If the pilot is successful, $500,000 will initially be invested in marketing and systems, and the product is expected to net $100,000 cash flow annually in perpetuity.

There is only a 50% likelihood that the pilot will be successful. If successful, the product will be put into production, and a normal level of risk would be expected. The normal discount rate used in your company is 10%. Should the company proceed with the pilot?

Solution 22.5:

If the pilot is successful (50% chance), the present value of the product will be

$$\$100,000 / 10\% – \$500,000 = \$500,000.$$

If the pilot is not successful, the present value of the product is zero. Before knowing whether the pilot will success, the present value of the product is $250,000. This present value is one year after the initial $100,000 outlay on the pilot project, whose net present value is $250,000 / 1.1 – $100,000 = $127,273.

Exercise 22.6: Options and Monopolists

On July 1, 2000, a pharmaceutical company is deciding whether to invest 3,000,000 in the development of a new drug. The company estimates that the probability of success is 5%, in which case it can obtain a patent and sell the drug as a monopolist beginning in year 2005. The marginal cost of production would be a constant 10 per prescription, and the quantity demanded (in millions of prescriptions per year) would be $q = 100p^{-1.5}$, where p is the price. After the end of year 2020, the patent would expire and there would be no future profits. The company's cost of capital is 10% and applies to all stages of the project. Assume that all cash flows for year z occur on July 1, z.

Solution 22.6:

We determine the expected profits if the drug is a success. The quantity demanded each year is $q = 100p^{-1.5}$, or $p = (0.01q)^{-b}$. The total revenue as a function of q (in millions) is

$$p \times q = (0.01q)^{-b} \times q = 0.01^{-b} \times q^{a}$$

The marginal revenue is $\partial(\text{total revenue}) / \partial q = a\ 0.01^{-b}\ q^{-b}$.

We set marginal revenue equal to marginal cost: $a\ 0.01^{-b}\ q^{-b} = 10$

$$q^{b} = a \times 0.1 \times 0.01^{-b} = 0.718$$
$$q = 0.718^{1.5} = 608{,}581.$$

To find the net profit, we find the price level as $p = (0.01q)^{-b} = (0.01 \times 0.608581)^{-b} = \$30.00$. (Here q is in millions.) The net profit is 608,581 × (\$30 – \$10) = \$12,171,620.

These are annual profits. The present value of a 16 year stream of such annual profits beginning in 5 years at a 10% cost of capital is

$$\$12{,}171{,}620 \times [\,(1 - v^{21}) - (1 - v^{5})\,] / (1 - v)$$
$$= \$12{,}171{,}620 \times (v^{5} - v^{21}) / (1 - v) = \$65.04 \text{ million}$$

The probability of success is 5%, so the expected profits are 5% × \$65.04 million = \$3.25 million. The initial investment is \$3 million, so the net present value of the project is \$3.25 million – \$3 million = \$0.25 million.

{This problem puts together microeconomics, interest theory, and corporate finance.}

Exercise 22.7: Real Options (Research and Development)

A firm is deciding whether to develop a new drug. Development costs are $1,400,000. Once developed, the drug must be approved before it can be sold. If the drug is approved, the company will incur marketing costs of $1,000,000. The probability is 75% that the drug will be a success. If the drug works well, the company receives a perpetual cash flow of $500,000 at the end of each year; otherwise, the company receives a net cash flow of $500,000 at the end of year 1, declining by $100,000 each year for the following four years, and no cash flow thereafter. Using an annual effective rate of 8%, calculate the probability of approval needed for the firm to develop the drug.

Solution 22.7:

For the firm to develop the drug, the present value of future cash inflows must be at least $1.4 million.

If the drug is approved, the subsequent investment is $1 million of marketing costs, and the expected cash inflows are

! 75% probability: $500,000 / 8% = $6.25 million
! 25% probability: $500,000 / 1.08 + $400,000 / 1.08^2 + $300,000 / 1.08^3 + $200,000 / 1.08^4 + $100,000 / 1.085 = $1.26 million.

Expected cash inflows = 75% × $6.25 million + 25% × $1.26 million = $5.00 million.

We subtract the marketing costs of $1 million to get a $4 million net present value. For the firm to develop the drug, it must expect that the probability of approval times $4 million exceeds $1.4 million, so the probability of approval must be at least $1.4 million / $4 million = 35%.

Question: How might one make this into a problem about real options?

Answer: In this exercise, the firm has no choice whether to continue production if the drug is not approved. To make this a problem with real options, we might give probabilities of approval of this firm's drug and of other firms' drugs, and have the probability of success contingent on whether this firm wins approval and whether other firms win approval of competing drugs. That makes the problem like real markets, not just a problem with real options.

Exercise 22.8: Trade Shows

To develop a new product, a firm must spend \$8 million in 20X6 and \$10 million in 20X7. If the firm is the first on the market with this product, it will earn \$40 million in 20X8. The firm believes it has a 50% chance of being the first on the market. In 20X9 and afterwards, many firms will have entered the market, and none will earn economic profits.

On January 1, 20X7, the firm participates in a trade show, where all the potential market entrants show their products. After the trade show, the firm will have a better estimate of its chance of being the first on the market. Its chance of being first on the market will be uniformly distributed over {0%, 100%}. The firm's cost of capital is 12%.

A. If there were no trade show, should the firm begin developing the product?
B. If there is a trade show, should the firm begin developing the product?

Solution 22.8:

Part A: If there is no trade show, the firm does not have the option to forego development of the new product after seeing the competition. The net present value of the project is

$$-\$8 \text{ million} - \$10 \text{ million} / 1.12 + 50\% \times \$40 \text{ million} / 1.12^2 = -\$0.98 < 0,$$

and the firm should not begin developing the project.

Part B: The firm will decide at the trade show whether to continue development. It continue development if the income expected in 20X8 exceeds the costs in 20X7. The costs in 20X6 are already expended when the trade show is held; they can't be recouped by stopping development of the product.

The expected income in 20X8, if the firm is first on the market, is \$40 million, or \$40 million / 1.12 = \$35.71 in 20X7 dollars. The cost to continue developing the product in 20X7 is \$10 million. Let p be the probability of being first on the market.

If $p \times \$40 \text{ million} / 1.12 > \$10 \text{ million}$, or $p > \$10 \text{ million} / (\$40 \text{ million} / 1.12) = 28\%$, the firm should continue development.

Before entering the trade show, the firm has a uniform probability distribution between 0% and 100% that it will be first on the market.

The probability is 28% that the firm will estimate its chance of being first on the market to be below 28%, in which case it should not continue development. It loses the \$8 million expended in 20X6.

The probability is 1 – 28% = 72% that the firm will estimate its chance of being first on the market to be above 28%, in which it should continue development. Its average chance of being first on the market (its contingent probability given that the probability exceeds 28%) is ½ × (28% + 100%) = 64%. The expected net present value of the project is

$$-\$8 \text{ million} - \$10 \text{ million} / 1.12 + 64\% \times \$40 \text{ million} / 1.12^2 = \$3.48 \text{ million}.$$

The real option in this exercise is the option to abandon development of the product once the firm learns its probability of being first on the market by viewing the competition at the trade show. The expected profit with the option to abandon is

$$72\% \times \$3.48 \text{ million} - 28\% \times \$8 \text{ million} = \$0.27 \text{ million}.$$

The value of the option to abandon is \$0.27 million – (–\$0.98 million) = \$1.25 million. This is a valuable option, and it turns the firm's decision from not developing to developing.

Business students sometimes wonder why firms have trade shows. They ask:

> Why give away competitive secrets? Don't other firms come to pry away competitive secrets? Why not wait until you have a finished product and then sell it to consumers? Don't this preserve your competitive advantages?

In fact, firms gain much from trade shows. They want to know which products they are likely to produce first, and they want to discourage other firms from producing the products that they are developing. Companies come to the trade show and revise their own development schedules in line with their competitors' progress.

! If a product is nearing completion by another company, a firm may forego further development and save money.
! If a firm can show that it is nearing completion of a product, other firms are likely to abandon further development, increasing the likelihood of success for the first firm.

The firms *sort themselves*, with each firm continuing development of the products which it is likely to be first to complete. Everyone gains from the trade show.

Question: This sounds like firms dividing the market among themselves.

Answer: This procedure relies on the free market and competition to stimulate the firms to innovate and to produce efficiently. If the firms do innovate and produce efficiently, this procedure enables them to reduce the waste of multiple firms competing until the last day.