

Corporate Finance, Module 23: "Advanced Option Valuation"

*Volatility Questions*

(The attached PDF file has better formatting.)

{This posting contains more information than is needed for the corporate finance on-line course.}

Question 23.1: Volatility

The stock of XYZ Company has an expected annual return of 10%. For the binomial tree pricing method, the upward movement  $u$  is modeled as  $e^{\sqrt{h}}$  and the downward movement  $d$  is modeled as  $1/u$ . If the downward movement of the stock is  $-15\%$  in the next six months and the annual risk-free rate is 6%, which of the following is closest to the standard deviation of the stock's annual return?

- A.  $(-\ln(0.85) / 0.5^{0.5})^{0.5}$
- B.  $-\ln(0.85) \times \sqrt{0.5}$
- C.  $\ln(0.85) / \sqrt{0.5}$
- D.  $-\ln(0.85) / \sqrt{0.5}$
- E.  $\ln(0.85) \times 0.5$

Answer 23.1: D

The downside change is

$$\begin{aligned} e^{-\sqrt{t}} &= 0.85 \\ -\sqrt{t} &= \ln(0.85) \\ -\sqrt{0.5} &= \ln(0.85) \\ &= -\ln(0.85) / \sqrt{0.5} \end{aligned}$$

This is a standard convention for choosing the upward and downward movements.

Question 23.2: Volatility

As the stock price volatility increases, which of the following is true?

- A. The call option value and put option value both increase.
- B. The call option value and put option value both decrease.
- C. The call option value increases and the put option value decreases.
- D. The call option value decreases and the put option value increases.
- E. None of A, B, C, or D is true.

Answer 23.2: A

Options are a one-sided investment: one gains if the stock moves one way, but one does not lose (except for the original premium, which is a sunk cost) if the stock moves the opposite way. More uncertainty in the stock price movement increases the value of the option, whether it is a call option or a put option.

### Question 23.3: Volatility

European call and put options are trading on a stock. The stock price is \$80, the strike price is \$80, the time to maturity is one year, and the risk-free interest rate is 6% with continuous compounding. As the stock price volatility increases without bound (i.e., goes to infinity), what happens to the value of the options?

- A. Call  $\rightarrow$  \$80; put  $\rightarrow$   $\$80e^{-6\%}$
- B. Call  $\rightarrow$  \$80; put  $\rightarrow$   $\$80 \times (1 - e^{-6\%})$
- C. Call  $\rightarrow$   $\$80 \times (1 - e^{-6\%})$ ; put  $\rightarrow$   $\$80e^{-6\%}$
- D. Call  $\rightarrow$   $\$80 \times (1 - e^{-6\%})$ ; put  $\rightarrow$   $\$80 \times (1 - e^{-6\%})$
- E. None of A, B, C, or D is true.

Answer 23.3: A

The call value  $\rightarrow$  \$80 and the put value  $\rightarrow$   $\$80e^{-6\%}$ .

In the Black-Scholes formula, as  $\sigma \rightarrow \infty$ ,  $d_1 \rightarrow \infty$  and  $d_2 \rightarrow -\infty$ , so  $N(d_1) \rightarrow 1$  and  $N(d_2) \rightarrow 0$ .

! The call option value  $\rightarrow S_0 \times 1 - PV(X) \times 0 = S_0$ .

! The put option value  $\rightarrow PV(X) \times (1 - 0) - S_0 \times (1 - 1) = PV(X)$ .

The maximum value of a call option is the current price of the stock, and the maximum value of a put option is the present value of the strike price. As the volatility increases without bound, the option approaches its maximum value.

Question 23.4: Volatility

One year European call and put options are traded on a stock whose stock price volatility is 30%. An investor is using a binomial tree pricing method with  $N$  periods to estimate the value of the options, with an upward stock price movement of  $e^{0.25 \times \sqrt{N}}$  and a downward price movement that is the reciprocal of the upward price movement. Which of the following is true?

- A. The investor has over-estimated the value of both the call and put options.
- B. The investor has under-estimated the value of both the call and put options.
- C. The investor has over-estimated the value of the call option and under-estimated the value of the put option.
- D. The investor has under-estimated the value of the call option and over-estimated the value of the put option.
- E. None of A, B, C, or D is true.

Answer 23.4: B

The volatility used in the binomial tree is too *low*, so the investor has under-estimated the value of both the call and put options.