Corporate Finance, Module 23: “Advanced Option Valuation”

Black-Scholes: Illustrative Test Questions

(The attached PDF file has better formatting.)

{This posting contains more information than is needed for the corporate finance on-line course.}

Exercise 23.1: Black-Scholes

Which of the following are true regarding the Black-Scholes model?

A. The Black-Scholes model assumes that stocks are continuously traded.
B. The Black-Scholes model assumes that a stock’s (unknown) future price has a lognormal distribution.
C. The Black-Scholes model can be adapted to value exactly options on short-duration bonds.

Solution 23.1:

Part A: The Black-Scholes model is a continuous time diffusion model. It assumes that stocks are traded continuously and that friction costs are not material. The Black-Scholes model is the binomial tree pricing method when the periods become infinitesimally short.

Part B: The Black-Scholes model assumes that a stock’s future price is now unknown but it can be modeled by a lognormal distribution.

Part C: Two problems with adapting Black-Scholes to value options on short-duration bonds are

1. The Black-Scholes model assumes that the risk-free interest rate is constant; options on bonds assume that the risk-free rate is stochastic. This is a hindrance to using Black-Scholes to value bond options. (Black-Scholes can be adapted as long as the term structure of interest rates is known and *not* stochastic.)

2. The Black-Scholes model assumes that the volatility is constant; or at least, that the volatility of the underlying security does not have a material drift. Stocks are assumed to have a relatively constant volatility, and changes in the volatility can not be foreseen. Bonds have a declining volatility: the market value of long-term bonds – but not short-term bonds – changes greatly with interest rates. The change in market value declines as the bond approaches maturity.

Question 23.2: Black-Scholes

The Black-Scholes formula can be used to value all but which of the following?

A. European put option on a stock index
B. European call option on a foreign currency exchange rate
C. European put option on a Treasury bill
D. European call option on a stock traded on a foreign exchange
E. The Black-Scholes model can be used to price all of A, B, C, and D.

Answer 23.2: C

The Black-Scholes formula assumes a constant value for the volatility parameter. The market value volatility of a Treasury bill decreases as the bill approaches maturity. Know that the Black-Scholes formula can not be used to price options on short duration bonds.

Exercise 23.3: Use of Black-Scholes

Which of the following are true?

A. The Black-Scholes method can be used to exactly value an American put.
B. The Black-Scholes method can be used to value an exchange rate option.
C. The Black-Scholes method can be used to price the value of a follow-on investment.

Solution 23.3:

Statement A: The Black-Scholes equation can not value exactly an option that might be exercised early. It is sometimes profitable to early an American put option early, whether or not the stock pays dividends, so the Black-Scholes equation can not value exactly an American put option.

Statement B: If the exchange rate option is European, so it can not be exercised early, it can be exactly valued by the Black-Scholes equation. For an exchange rate option, we need both the domestic and the foreign risk-free interest rate; otherwise the exchange rate option is like any other option.

Statement C: This statement can be answered as either true or false. In general, the Black-Scholes equation can be used to value real options just as it is used to value financial options, so it can be used to value a follow-on investment. In practice, we need the volatility of the returns of the follow-on investment, and we must assume that the return from the investment has a lognormal distribution. The answer is true, with caveats.

Question 23.4: Distribution

{*Note:* This illustrative test question covers material not on the syllabus for this course. Many candidates hear about the Black-Scholes assumptions and what to know what they are, even if it is not needed for the final exam. We give the answer, without showing the derivation.}

A Black-Scholes model with parameters σ, r, and t (for volatility, risk-free rate, and time to maturity) assumes which of the following?

A. The stock price *rate of return* is *normally* distributed with mean rt and standard deviation $\sigma\sqrt{t}$.
B. The stock price *rate of return* is *lognormally* distributed with mean rt and standard deviation $\sigma\sqrt{t}$.
C. The stock price *return* is *normally* distributed with mean rt and standard deviation $\sigma\sqrt{t}$.
D. The stock price *return* is *lognormally* distributed with mean rt and standard deviation $\sigma\sqrt{t}$.
E. None of A, B, C, or D is true.

Answer 23.4: E

The correct answer would be that the stock price *rate of return* is *normally* distributed with mean $r - \sigma^2 t/2$ and standard deviation $\sigma\sqrt{t}$ in a *risk-neutral world*. In the real world, the stock price rate of return is normally distributed with mean $\mu - \sigma^2 t/2$ and standard deviation $\sigma\sqrt{t}$. The Black-Scholes model is more complex than seems at first glance.