Corporate Finance, Module 23: “Advanced Option Valuation”

*Homework Assignment*

(The attached PDF file has better formatting.)

*Black-Scholes Pricing*

A stock’s price volatility is 35%. The annual effective risk-free rate is 8%. A non-dividend paying stock now trades at $80. Six-month European calls and puts are trading with a strike price of $85.

A. What are the values of the five input parameters to the Black-Scholes model for pricing these call and put options? (The *sixth* input parameter, the dividends, is not considered in this exercise.)
B. What is the value of PV(X): the present value of the exercise price? The interest rate uses annual compounding.
C. What is the value of S ÷ PV(X): the ratio of the stock price to the present value of the exercise price?
D. What is the value of *ln*(S/PV(X)): the logarithm of the ratio of the stock price to the present value of the exercise price?
E. What are the values of $d_1$ and $d_2$? (Work out $d_1$ by formula and $d_2$ from $d_1$.)
F. What are the values of $N(d_1)$, $N(–d_1)$, $N(d_2)$, and $N(–d_2)$? Use either a cumulative normal distribution table or a built-in spread-sheet function. The textbook has a cumulative normal distribution table; Excel has a built-in function.
G. What is the value of the European call option? (Use the formula for a call option.)
H. What is the value of the European put option? (Use the formula for a put option; you will use put call parity in the next question.)
I. Verify that the put call parity relation holds.

The values of $d_1$ and $d_2$ are shown below. You will be given these formulas on the final exam, but you must know how to use them to derive put and call option prices.

$$d_1 = \frac{\ln(S / PV(X)) + (\sigma^2 / 2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S / PV(X)) - (\sigma^2 / 2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

*Question:* For Part A, should we know the five inputs for the final exam?

*Answer:* Know the five inputs and know also which are stated in the options contract (strike price, time to maturity), which are known financial items (risk-free interest rate, stock price), and which are estimates (volatility).

*Question:* For Part B, do we use continuous compounding or annual compounding for the Black-Scholes formula?

*Answer:* We should use continuous compounding. The formula is exact if prices change continuously. To keep the mathematics simple enough for the average college student, Brealey and Myers use annual compounding.

*Question:* Several candidates ask on the discussion forum whether the stock price volatility is the σ in the Black-Scholes formula or the $\sigma^2$. Is the volatility the standard deviation or the variance?

*Answer:* The volatility is the $\sigma$. This is the standard deviation *per square root of the unit of time*. $\sigma^2$ is the *variance rate*, or the variance *per unit of time*.

*Question:* What does *standard deviation per square root of the unit of time* mean? This is the standard deviation of the stock price; what does it have to do with time?

*Answer:* The stock price is a scalar, not a random variable.

- A random variable has a distribution, which has a standard deviation.
- A stock price is a single value (a scalar); it does not have a standard deviation.

*Question:* The homework assignment says the stock price volatility is 35%. The volatility refers to the stock price; it makes no mention of time.

*Answer:* The stock price now is known. The stock price one year from now is not known with certainty.

- If we know the current stock price and the expected return, we know the expected stock price one year from now, which is the mean of the stock price distribution in one year.
- If we know the volatility of the stock price, we know the standard deviation of the stock price distribution in one year.

*Question:* One year from now, the stock price will be a scalar, just like it is a scalar now. If the stock price now does not have a standard deviation, why does it have a standard deviation one year from now?

*Answer:* To be rigorous, we should say: "The distribution of the possible stock prices one year from now." The stock price now is $80. In one year, the stock price may be $50, $85, $150, or some other figure. Each possible stock price has a likelihood. The likelihoods form a probability density function (pdf). The pdf is a lognormal distribution, whose parameters depend on the stock price now, the expected return, and the volatility.

*Question:* If the volatility is 35%, is the standard deviation of the stock price distribution in one year 35%?

*Answer:* The stock price in one year has a lognormal distribution, which is *e* raised to the power of a normal distribution. The normal distribution in the exponent of *e* has a standard deviation of 35%. The standard deviation of the lognormal distribution itself is a complex expression of $\mu$ and $\sigma$.

*Question:* Is the standard deviation of the stock price in half a year = ½ × 35% = 17.5%?

*Answer:* If the variance in a year is $35\%^2$ = 12.25%, the variance in half a year is ½ × 12.25% = 6.125%. The standard deviation in half a year is $6.125\%^{0.5}$ = 24.749%. The volatility is the standard deviation *per square root of the unit of time*.

*Question:* Is this material covered in the corporate finance course?

*Answer:* This material is covered on the actuarial exams. We mention it here because some candidates asked about it on the discussion forum.