

Corporate Finance, Advanced Options, practice problems

** Exercise 23.1: Real Options

You give a presentation on real options, and one member of the audience asks why we can't price the real option by a discounted cash flow procedure as we price other projects.

Explain whether each of the following responses is correct.

- A. The cash flows from the real option are stochastic; we must have deterministic cash flows to use discounted cash flow techniques.
- B. Investors are risk-averse; discounted cash flow techniques are useful only for risk-neutral investors.
- C. The capitalization rate for the real option changes stochastically over the life of the project; we need known capitalization rates for discounted cash flow techniques.
- D. Option pricing gives lower net present values and allows more conservative pricing.
- E. Option pricing gives higher net present values and allows us to sell more options.

Answer 23.1: C

Statement A: False. We use discounted cash flow techniques for stock prices, business projects, insurance policies, and other securities with stochastic cash flows. We use the expected cash flows; we need not know the future cash flows with certainty.

Statement B: False. The traders (investors) in stocks, business projects, insurance policies, and most securities that we price with discounted cash flow techniques are risk averse. We use a risk adjustment capitalization rate based on the systematic risk of the security.

Statement C: True. The systematic risk of the option is a multiple of the systematic risk of the underlying security. When the option is in-the-money or out-of-the-money, the multiple is low. When the option is at-the-money, the multiple is high. As the stock price moves, the multiple changes, so the capitalization rate for the option changes.

Statements D and E: False. Option pricing gives market values; it is neither conservative nor aggressive.

**** Exercise 23.2: Options Pricing and Risk-Free Rates**

Option pricing using the Black-Scholes model and the binomial tree pricing method use risk-free rates instead of the market capitalization rate for the underlying security.

Explain which of the following is the rationale for using the risk-free rate.

- A. Using the risk-free rate in combination with the stock price volatility is the same as using the stock's capitalization rate.
- B. Most investors in options are private investors who are risk-loving.
- C. For short-term options (three months or less), the difference between the risk-free rate and the market capitalization rate is not material.
- D. These methods price a risk-free portfolio of the option plus or minus some shares of stock; the risk-free portfolio should earn a risk-free return.
- E. Options traders hedge their risks and can afford to offer the options at the risk-free rate.

Answer 23.2: D

Statement A: The stock's capitalization rate is determined from the risk-free rate and the stock's *systematic risk*, not its total volatility. Options pricing uses total volatility, not only the systematic risk of the stock. The statement makes no sense; it is not just wrong, it is meaningless.

Statement B: Many types of investors buy options. Some institutional investors buy options to hedge risks. Some private investors are attracted to options as a form of gambling, but not all private investors (or even most investors) in options are risk-loving. Financial economists assume investors are risk averse, whether they trade stocks, bonds, or derivatives. The pricing of options give market prices; it does not depend on the risk aversion of the investor.

Statement C: The difference between the average market capitalization rate and the risk-free rate is the market risk premium, which is about 7% to 8%, or about 2% for a three month term. The market value of the underlying securities on which options are traded is many trillions of dollars, so the difference in the interest rates is tens of billions of dollars.

Note: we often use risk-free rates to value dividends on shares before the option expires. The actual rate should be slightly higher, but the difference is so small (a few cents at most) that it is ignored. For the option itself, not just the dividend on the stock paying a dividend shortly before the option expires, we use the proper discount rates.

Statement D: If the option delta is Δ , the option price changes by Δ when the stock price changes by \$1. The combination $\Delta \times \text{stock price} - \text{option price}$ does not change when the stock price changes, so it is risk-free. The Black-Scholes formula prices the combination $\Delta \times \text{stock price} - \text{option price}$, so it uses the risk-free rate.

Statement E: Options are very risky; the return can be very high or very low. Banks or other institutions which sell options to private investors often hedge their risks, so they have relatively risk-free option portfolios on which they collect sales margins and brokerage fees. This has nothing to do with the use of the risk-free rate to price the option.